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Parametric Design Optimization of CFRP Cable-based Beam–String Structures using a Reproducible Reduced-Order Numerical Model

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ABSTRACT: Incorporation of the factors of initial geometry, cable axial rigidity, prestressing force, and serviceability constraints is essential for optimizing design of CFRP cable-based BSS structures. This paper documents the development of a computational workflow for evaluating the interaction of the rise-to-span ratio, cable cross-sectional area, and prestress. The workflow uses an Euler-Bernoulli beam finite element model, and represents the "contribution of inclined cable-strut segments with two equivalent vertical tangent springs. The workflow is parameterized for a range of rise-to-span ratios (0.05 to 0.15), cable cross-sectional areas (400 to 1,200 mm²), and prestressing force levels (100 to 300 kN). For each parameter case, the workflow calculates the mid-span deflection of the modeled beam, the prestressing force level, the spring constant of the equivalent spring, the length of the cable segment, and a proxy for the volume of the cable segment. To compare different designs, a score is calculated, which represents a trade-off between the proxy for the volume of the cable segment and the mid-span deflection of the beam. For the case study illustrated in this paper, it is assumed that the mid-span deflection of the beam should be less than span/300. If the mid-span deflection does not exceed 300 MPa, up to 858 candidate cases were found to satisfy the computational constraints. Based on the case study, the constraint score was the lowest for a rise-to-span ratio of 0.15, cross-sectional area of 400 mm² cable, and a prestressing force level of 225 kN. The nominal prestressing force level was 562.5 MPa, and the mid-span deflection was 10.05 mm. The results of the case study should be interpreted as a conditional optimum based on the assumptions of the computational model. The paper describes a methodology for initial-value parameter screening that is ideal for identifying and overcoming computational roadblocks related to nonlinear equilibriums, specifically cable-strut structures.

1. INTRODUCTION

Long-span structures require stiffness, ductility, strength, and stability. In addition, they should be constructed efficiently and effectively. In this respect, beam-string structures (BSSs) are particularly attractive, since they comprise a beam that supports bending and compression, a tension string, and a set of struts that link the two subsystems. In BSSs, initial equilibrium is achieved through prestressing, which creates camber, alters the distribution of internal actions and stiffness, and affects the position of the neutral axis. It is the integration of all the above mentioned factors that makes BSSs systems unmanageable by analytical means and requires numerical simulation for their design.

In structures where a steel tension element is used, a CFRP element may be used as an alternative, as it is corrosion-resistant and has a high long-term strength-to-weight ratio. CFRP tendons may be modeled as cables. Similar to steel, the effective "elastic modulus and other material properties of a CFRP tendon depend on the fiber architecture and the resin system. A structure may be extremely stiff, or may have a small self-weight due to the use of lightweight materials. Compared to other systems, the most critical and difficult aspects of CFRP cable structures are anchoring and integrating the CFRP system to the main structure.

A challenge in design is balancing the effects of rise-to-span ratio, cross-sectional area of cables, and prestress. Generally, using a greater rise increases the vertical component of the action and alters the path of the force. Increasing rise also makes the cable system longer and adds architectural depth. An increase in cable area, besides increasing the mass and anchorage, also contributes to increased rigidity, which results in less deformation. Increasing prestress creates a preloaded state and may alleviate some load-induced deflection. Too much prestress increases compression and load in the support, increases camber, and increases the demand on anchorages. A balance of the three parameters may be achieved at the expense of both safety and efficiency. Resource usage and response should be optimized simultaneously. Xue and Liu (2009) optimized and studied the behavior of a BSS under the influence of the aforementioned variables, while Xue and Liu (2008) studied a large-span pipeline crossing.

This foundation notwithstanding, a good design study should seek to make an optimization problem reproducible. There are numerous reasons that simply stating "increase prestress" is unsatisfactory. Similar remarks apply to the interpretation of optimization results. A result should always be considered in the context of the mechanical systems it represents and the degrees of freedom of the model. Transparent computational methods, represented by equations and reported along with the assumptions that govern a computational analysis, should enable evaluation of the calculations. This is particularly important for establishing a reproducible method to evaluate design variables in a construction-ready CFRP BSS. This work is not to the replace the work of a detailed project analysis. This study aims to provide a reliable methodology to assess candidate design variables, and offer a framework within which engineering judgment may be used to evaluate these variables.

The study contains four elements. The first is to study how the ratio of rise to span, the area of the cable, and the prestressing force, affect the calculated mid-span deflection and the equivalence of the cable system. The second is to study the compromise between deflection and a proxy for cable volume, represented by the design score. The third is to study the dominating effect and interactions of other parameters present in the tested system. The final element of the study is to formulate and interpret the results in the context of design, and provide a protocol that is generally useful in designing cable systems. The questions asked in the study are as follows. Which of the three parameters dominates mid-span deflection in the case study? How do area and prestress affect the cable system? Which design is preferred among a set of candidates based on design score? What hypotheses are drawn from the case studies? How generally valid are the results?

The approach and study protocol are the main contributions of this research. It contains a detailed analysis of 891 case studies rather than reporting an unreported analysis or experiment. The results are case study output rather than prototype output. This paper reports evidence of design case studies rather than testing a hypothesis. Published research is analyzed to define the variables of the case studies rather than to provide evidence to support the case studies. Research is performed to validate the case studies.

2. LITERATURE REVIEW AND DESIGN CONTEXT

2.1 Structural behaviour and the role of prestress.

A BSS derives its behavior from a self-equilibrated force system. The principle of Balance requires that the initial cable force be increased to provide camber and to counteract the actions of the beams and struts. Under service load conditions, the actions of the components of the force system are interchanged by means of geometry. This defines the level of prestress. However, a prestressing force of the entire capacity of the cable may cause the cable to fail by excess constrain. It is essential that some safety and serviceability limits be considered. Xue and Liu (2009) described that the proper level of prestress is that which puts the system in equilibrium, and considered the limits of serviceability, while considering ultimate load conditions.

2.2 Rise-to-span ratio.

Changes in the rise-to-span ratio modify the shape of the cable system. Cable length and the horizontal and vertical components of cable action change. In a simplified approach, in which the cable system is represented by a set of (tangent) springs, the vertical stiffness of the system changes with the rise-to-span ratio. In addition, the location of the cable struts and the continuity of supporting beams and other frame members affect the components of the horizontal and vertical cable actions. Rise-to-span ratio and strut arrangement influence the stability and performance of structural systems. Other parameters, such as restraints and supports, define the character of the structural system. Xue and Liu (2008) stated the same in the context of systems of long span beam-string structures. This research suggests that a geometric ratio cannot be considered in isolation to determine an optimum.

2.3 Cable area and axial rigidity.

The axial rigidity EA is a first-order model of a tension element's elastic behavior. Larger areas and moduli of tension elements result in lower tensile strain. However, only a consideration of the structure/cable geometry through which a cable force acts on a beam results in a rigorous analysis of tensile behavior. Area and axial rigidity should be considered on an equal basis. Liu et al. (2015), and Yang et al. (2020) outline a number of differences between CFRP and steel cables, including the mechanical behavior of cables, configuration of the cables, and impacts and interactions of cables. Given the aforementioned differences, it can be inferred that cable area optimization should be done judiciously. The definition of a cable by its nominal area provides little to no information regarding the tensile behavior of the cable, including its modulus, fatigue behavior, and its interaction with other materials. It can be assumed that CFRP attains its tensile strength limit prior to failure.

2.4 Numerical and experimental optimization of BSS.

The literature provides examples of combining analysis and experiments. Xue and Liu (2009) used ANSYS, design optimization, three types of scale model tests, and a large-deformation non-linear analysis. Xue and Liu (2008) performed a numeric study on a 100 m beam-string pipeline crossing using modeling and analysis of non-linear buckling under different boundary conditions. Lee et al. (2019) performed a numerical and experimental analysis of a two-way Boot Strapping System (BSS) and showed the importance of the cable system's ability to support both positive and negative loads. The studies described above addressed geometry, prestress, and cable position as integrated design variables. Thermo-mechanical experiments and analysis have shown environmental and loading conditions affect BSS behavior and response. However, for the present study of a static grid, these conditions are not taken into account (Qi et al., 2023). The studies did not determine a general design case for CFRP BSS that would apply across the board.

2.5 CFRP-specific considerations.

Directional properties of CFRP result in phenomena specific to it (e.g. tendon splitting and anchorage crush) that can be appreciably different from other cable materials. For a CFRP cable element modeled using a link/bar element, tensile stiffness can be achieved provided the effective axial modulus, tensile strength, and cross-sectional area of the element are specified. Such a model cannot account for transverse crash failures and contact pressures. Han et al. (2017) and Wang et al. (2018) noted that the configuration of CFRP tendon anchorage governs its use in cable-stayed and Balanced stay bridge structures. Therefore, in global analysis, the decision of the cable area should be treated as conditional, and the safety of local connection should not be inferred by calculating beam deflection.

2.6 Optimization methods and transparent objectives.

Structural optimization relies on the specification of design variables, objective functions, constraints and parametric bounds. Single objective design can result in seemingly sub-optimal solutions. For example, using the smallest cable to minimize structure weight may result in excess cable tension, unnecessary dead load and reduced constructability of the system. A larger cable may be needed to ensure acceptable stress and strain states. Using multiple objective functions allows the designer to explore various trade-offs. The users of this paper are able to express preferences to reduce deformation. To facilitate this, various solutions were formed based on an (arbitrarily) weighted sum of deformation and a surrogate volume of the cable. In addition, the algorithm was constrained from selecting a solution that resulted in excess cable dead load and a strain state outside acceptable limits. From the constraints and design variables specified, the algorithms interpreted design trade-offs appropriately (Deb, 2001; Bendsøe & Sigmund, 2003).

2.7 Research gap.

Prestress, rise, and beam section, as well as cable and strut areas, have been found to be critical in BSS studies. Additionally, the studies on the use of CFRP in BSSs have identified the direction and the adequacy of anchorage of CFRP as critical. Previously, the authors established the effects of variable direction on CFRP tension cables in BSSs. A practical research gap remains in presenting a clear, screenable, parameter-based approach to BSSs that incorporate CFRP as one of the constituent tensioning systems. The current work will not be the first to optimize BSSs; the main goal will be to show a reasonable, clear case-specific evaluation of the concerned variables.

3. RESEARCH DESIGN AND OPTIMIZATION FORMULATION

3.1 Study design.

The research is a deterministic numerical parameter study. A fixed beam model has been developed and the scope of the research is to understand the effects of rise-to-span ratio (r), cable sectional area (A_c), and a prestressing force (T_0). The effects of the remaining parameters including the span and section of the beam, type and location of supports, type and location of distributed loading, and type and size of CFRP, have been fixed for the studies presented herein. This permits the effects of the three variables under study to be isolated. The variable r has been set at 0.05, 0.10 and 0.15; A_c has been set at 400, 800, and 1200 mm^2^; T_0 has been set at 100, 200, and 300 kN. There are 11 x 9 x 9 = 891 cases.

3.2 Design variables.

The rise-to-span ratio is given by r = h/L, where L is the span of the beam and h is a nominal rise for defining the equivalent cable system. The more the cable is stretched the higher the stiffness, i.e. the more rigid the system. The more the cable is stretched, the higher the force in the cable. This force contributes to the stiffness of the system, and thus affects the deflection of the cable. These parameters are considered as the major design parameters of the BSS. Earlier studies in BSS optimization consider similar parameters (Xue and Liu, 2009, 2008). Our current model does not consider optimization of the beam cross-section, and we hold the strut spacing constant. This is to study the effect of the other design variables considered in the model.

3.3 Objective function.

We consider a trade-off between an estimated quantity of cable, and the estimated mid-span deflection of the cable. An estimated quantity of cable, Vp, is considered. This is calculated as the cross-sectional area of the cable multiplied by an approximate length of four cable segments. Embodied carbon and cost are not considered. Let Vmin and Vmax be the estimated quantity of cable among the allowed designs, and let δmin and δmax be the mid span deflection for the respective designs. The score is given by J = 0.5 ((Vp - Vmin) / (Vmax - Vmin)) + 0.5 ((δ - δmin) / (δmax - δmin)). Smaller values of J are desired. It is clear from the above analysis that equal weights are used to simplify the analysis. This is not an indication that the two components are of equal economic importance. Other components could be given different weights. The nature of the project may dictate retaining a Pareto optimal set as opposed to selecting a single solution.

3.4 Constraints.

Two screening constraints are applied. First, the calculated absolute midspan deflection must not exceed L/300, which is 40 mm for the example with a span of 12 m. This constraint is used for the example to illustrate an analytical screening limit, and is not to be construed to represent an applicable code requirement for all BSSs and roof systems. Second, the nominal prestress, σ_0 = T_0/A_c, must not exceed 600 MPa. This constraint is an example of a scenario value, and for project design, this value must be replaced by the product's allowable stress, along with the applicable resistance and code factors. The constraints are intentionally broad, to enable the user to recalculate the limits with alternate acceptance criteria.

3.5 Response measures.

The model provides, among others, measurements of midspan deflection, nominal prestress stress, an approximate value for equivalent vertical springs and the length of the cables. It provides a volume proxy of cables. The thrust of this example is to provide a rationale for the material usage and to provide a serviceability check. The constraints of the model are such that it approximates the nominal prestress, and therefore a complete check of stress of the component is outside the scope of this example. The equivalent springs and cables are approximations, and therefore, direct assessments of physical segments of cables or struts are outside the scope of this example. Nonlinear behaviors are outside the scope of this example.

3.6 Search procedure and reproducibility.

The full factorial design is enumerated instead of relying on a opaque optimizer. Candidate solutions to the FEM are evaluated to determine constraints. The normalization and weights are applied to evaluate the feasible solutions. The design selects the option with the highest score. The small size of the design space and a total of 891 solutions warrant the use of grid search. Grid search provides the opportunity to evaluate the search process to ensure stochastic effects are reduced. Grid search is appropriate for small, continuous spaces. Differential Evolution, for example, is appropriate for larger, more continuous, design spaces (Storn & Price, 1997; Virtanen et al., 2020). For this case, an optimizer would not address the inadequacy of the structural model to represent the design domain or function to capture design objectives of the system.

3.7 Evidence discipline.

Synthetic data were not used. Likewise, model parameters were not set by fabricated data. The results are not claimed to emulate or correspond to an ANSYS model. The rationale for the choice of design variables is supported by literature, and the interpretations are made within the context of mechanical engineering. Other works, both recent and relevant, were considered during the formulation of the design, such as the recent numerical and experimental studies of CFRP cable-truss systems (Han et al., 2026). It is assumed that the corresponding code, based on the articulated design and assumptions, produces outputs relevant to the design. Literature and published studies suggest a pathway to experimentally validate the results of the design. It is, however, assumed that the literature studies would reflect design needs and outcomes.

4. REDUCED-ORDER FINITE-ELEMENT MODEL

4.1 Structural idealization.

The upper beam is divided into 48 Euler-Bernoulli beam elements, yielding a total of 49 nodes. Each node is represented by vertical displacement and cross-section rotation degrees of freedom. Both ends of the beam are supported in the vertical direction with rotations unrestrained. A uniform vertical load of q = 10 kN/m acts along the length of the beam. The modulus of elasticity of the material of the beam is E_b = 200 GPa and its second moment of area is I_b = 8.0×10^{-4} m^4. Using these parameters, a reference problem is formed.

4.2 Beam element.

For an element of length l_e, the local Euler-Bernoulli bending stiffness matrix is given by k_e = E_b I_b/l_e³ × M_e, where M_e is the standard 4 × 4 matrix containing terms 12, 6l_e, 4l_e², and 2l_e². Matrix M_e is formed using standard procedures of finite element method of structural mechanics (Bathe, 2006; Cook et al., 2002; Zienkiewicz et al., 2013). The global stiffness matrix is populated with local beam element stiffness matrices by integrating M_e over the element. A uniformly distributed load acting over the element is modeled using a load vector, where the components of the load vector are ql_e/2 with respect to the first and fourth positions, and the components of the remaining positions are 0. It is to be noted that vertical displacements of the ends of the element are constrained. The linear system of equations is solved using a direct linear solver. The stiffness matrix of the system is symmetric.

4.3 Equivalent cable-system springs.

The cable-strut subsystem is modeled by two vertical, equivalent, tangent springs, located at quarter- and three-quarter-spans of the beam. This model is a gross approximation and should not be misconstrued to represent an exact representation of cable and strut anchorage. The horizontal distance of both springs from the neutral axis is taken to be L/4, and their vertical distance is taken to be 0.75h. Segment length, l_c, is evaluated using the formula given above. The direction cosines in the vertical and horizontal plane are evaluated to be 0.75h/l_c and L/4/l_c respectively. The axial, material, and geometric, contributions to the vertical stiffness are represented by k_v = E_c/l_c c_v^2 + T_0/l_c c_h^2. The first term represents the axial material stiffness and the second term is the geometric effect of prestressing. It is to be noted that, to capture the vertical stiffness, effect of both geometric and material rigidity are to be considered.

4.4 Material assumptions.

Through the entire set of variability cases, the modulus of CFRP is held to E_c = 150 GPa. This does not reflect a property of a real world component. No real density of the material is considered; thus, conclusions drawn regarding the system should not include mass considerations. A detailed analysis of the system, considering the above restrictions, should be conducted by a

practicing engineer to arrive at realistic conclusions. The variability of the axial rigidity of the element is considered by changing the modulus of the material and not of a system component.

Table 1. Model parameter ranges and fixed assumptions

Parameter	Value / range	Role
Span L	12 m	Fixed geometry
Beam E_b	200 GPa	Fixed material assumption
Beam I_b	8.0×10^{-4} m^4	Fixed section assumption
Uniform load q	10 kN/m	Fixed load case
Rise-to-span r	0.05–0.15; step 0.01	Optimization variable
Cable area A_c	400–1,200 mm^2; step 100	Optimization variable
Prestress T_0	100–300 kN; step 25	Optimization variable
CFRP modulus E_c	150 GPa	Representative assumption
Deflection screening	L/300 = 40 mm	Illustrative constraint
Nominal stress screening	600 MPa	Scenario constraint

4.5 Model outputs.

The main output shows the absolute vertical displacement at midspan of the beam. The nominal prestressing stress is calculated as σ0 = T0/Ac and reported in MPa. Equivalent spring constants are reported in MN/m. The approximate length of the cables is taken as four times the distance lc, and Ac is used to compute a volume proxy for the cables. The length of the cables has been approximated using a geometric construct, and is not an exact representation of the cables in the structure.

4.6 Verification checks.

Since candidates are evaluated with a constant 48 element mesh, differences in results are not due to mesh size. Elements are constrained to represent supports of the beam. The stiffness matrix is created and the corresponding analysis is performed on all candidates. According to the author, all candidates in the stated range yielded a finite solution. The solution was evaluated to provide qualitative results in agreement with the model. In the case of increasing spring stiffness, it was expected that midspan deflection of the beam should decrease. The variation of prestressing stress was expected to be inversely proportional to the cross-sectional area. This was a consistency check and should not be construed as an independent verification. Full geometry and mesh refinement are expected to provide better verification.

4.7 Scope of model interpretation.

This model is linear and reduced-order. Among the many simplifications in the model are: geometric nonlinearities in the complete beam-cable equilbrium, the presence of cable elements represented by tension only, sag of cables and redistribution of cable forces, the shape and deformation of individual cable and strut elements, out-of-plane responses, initial geometrical imperfections of the structure, and yielding of the beams. Together, these justify consideration of results from the model only for early-stage screening of potential designs, and not for assessment of final resistance or stability. This model has been explicitly developed for early-stage screening of design options, and extension of the model to incorporate the additional physics is described in the section on future work.

5. PARAMETRIC RESULTS

5.1 Grid overview.

Results of a full factorial assessment would consider 891 different designs. Of these, 858 designs (~95%) were determined to be feasible under the following limit conditions. 1) The midspan deflection of the deck is less than or equal to 40 mm, and 2)

the nominal prestressing tensile stress is less than or equal to 600 MPa. Of the 33 designs that were determined to be infeasible, 29 of these designs yielded unacceptable nominal prestressing tensile stresses of cables of less area than the larger of the two cables. The remaining 4 designs violated both limit conditions. It should be emphasized that the high feasibility rate does not prove that a majority of real-world BSS structures are safe. No analysis of the limits of strength, stiffness, and/or non-linear stability were performed. The values of rise-to-span ratios of 0.075 and 0.125 shown in Table 2 represent further sensitivity evaluations, and are not optimization steps.

Table 2. Selected sensitivity cases

Case	r	Area (mm^2)	Prestress (kN)	Deflection (mm)
Rise sweep	0.05	800	200	14.0017
Rise sweep	0.075	800	200	11.8473
Rise sweep	0.1	800	200	9.9269
Rise sweep	0.125	800	200	8.4023
Rise sweep	0.15	800	200	7.25
Area sweep	0.1	400	200	12.4183
Area sweep	0.1	600	200	11.0293
Area sweep	0.1	800	200	9.9269
Area sweep	0.1	1000	200	9.0307
Area sweep	0.1	1200	200	8.2877
Prestress sweep	0.1	800	100	9.9637
Prestress sweep	0.1	800	150	9.9452
Prestress sweep	0.1	800	200	9.9269
Prestress sweep	0.1	800	250	9.9086
Prestress sweep	0.1	800	300	9.8904

5.2 Effect of rise-to-span ratio.

At a cable area of 800 mm^2 and a prestress of 200 kN, the calculated midspan deflection is approximately 14 mm when the rise-to-span ratio is 0.05. Decreasing the ratio to 0.15 reduces the calculated deflection to approximately 7.25 mm. Deflection corresponding to rise-to-span ratios of 0.075 and 0.10 is approximately 11.85 mm and 9.93 mm, respectively. Deflection corresponding to a rise-to-span ratio of 0.125 is approximately 8.40 mm. The calculated deflection decreases monotonically with an increase in the rise-to-span ratio due to the geometry of the spring. With an increase in rise, the material contributes more in the vertical direction due to the larger vertical direction cosine. The approximate cable length for a rise-to-span ratio of 0.05 is 12.13 m, and for a rise-to-span ratio of 0.15 is 13.16 m. From the above, it can be stated that rise is a variable that needs to be traded off with deflection rather than increased to minimize deflection.

5.3 Effect of cable area.

At r = 0.10 and T_0 = 200 kN, predicted deflection is reduced from about 12.42 mm to 8.29 mm as cable area is increased from 400 to 1,200 mm2. Deflection is estimated at 11.03 mm, 9.93 mm and 9.03 mm for cable areas of 600, 800 and 1,000 mm2, respectively. For this case the effect of area on spring stiffness is significant, and the effect of the beam stiffness is negligible. Nominal prestress stress is reduced from 500 MPa for a cable area of 400 mm2 to 166.7 MPa for a cable area of 1,200 mm2. For all considerations of this model, increased cable area reduces nominal prestress stress and deflection, however

the increase is not fully beneficial and has to be offset against increased material use. It should be noted that reduction in material use and weight of the cables is not included in this study.

5.4 Effect of prestress.

At r = 0.10 and A_c = 800 mm², calculated deflection at a given prestress level can be estimated using the typical order of magnitude analysis. With prestress of 100 to 300 kN, calculated deflection varies from about 9.96 to 9.89 mm, respectively. Hence, within the range of the prestress level considered, the area effect is the predominant effect. This is because the stiffness of the beam in bending is limited, and within this range the effect of the prestress contribution to the equivalent spring constant is small. Nominal prestress increases in a linear manner from 125 to 375 MPa. Prestress is an important feature of this model. However, in a structure with a geometrically nonlinear behavior, prestress can significantly affect the system behavior, equilibrium state and stiffness. The model should not be used to exclude effects of prestress in an actual structure. A more detailed analysis may be warranted with the model.

5.5 Interaction between area and prestress.

Area and prestress combined govern the nominal stress ratio T_0/A_c. On the other hand, for a given prestress, the larger the area, the smaller the nominal stress. Larger prestress, for a given area, results in higher nominal stress and tends to increase the prestress-related spring stiffness. Thus, from the viewpoint of optimization, prestress should not always be viewed as a positive variable. The feasible design space removes combinations of prestress and area that exceed the designated threshold stress. Of the optimized designs, the lowest area/prestress combination (400 mm² and 225 kN respectively) results in a nominal stress of 562.5 MPa, which, according to the design threshold, is allowable. From the above, it can be seen that, in practice, design variables should be reported with their margin constraints.

5.6 Interaction between rise and area.

Rise corresponds to the change in the cable axis projection to the vertical, and area corresponds to the change in the axial rigidity. Therefore, combinations of rise and area may result in similar deflections. Deflections that appear similar may result from design decisions that are not structurally related. These combinations differ in terms of physical constraints, such as cable length and stress, the components and geometry of cable force and anchorage, architectural depth, and the design intent. While assessing combinations of rise and area, users are instructed to consider architectural constraints and the trade-offs that affect design decisions, such as the cost of material and the deformation allowed by the structure. Rise and cable area should be kept as independent variables for early design decisions.

5.7 Interaction among all three variables.

Best candidate in the grid is r = 0.15 with A_c = 400 mm², and T_0 = 225 kN. With these input values, the model calculates a midspan deflection of 10.05 mm, an approx. nominal prestress of 562.5 MPa, an approx. cable stiffness of 3.13 MN/m, an approx. cable length of 13.16 m, and a volume proxy of 0.00526 m³. This candidate is not the model's best deflection performer. The model's best performer is r = 0.15 with a cable area of A_c = 1200 mm², and T_0 = 300 kN, which results in a calculated deflection of 5.69 mm. The larger cable area makes the 2nd candidate more appropriate for a design using a multi-criterion equal-weighted objective. The difference in results illustrates the main purpose of multi-criterion design. That is, the result of the compromise shown is different from the outcome where one design constraint is eliminated.

5.8 Sensitivity ranking within the tested range

Comparing solutions for a single variable at a time, the effect of rise-to-span ratio on deflection was the most significant of those observed within the tested range, followed by the effect of cable area, and the effect of prestressing force. This effect was observed for the considered linear simplified model of the BSS. Variations to the geometry and load conditions of the beam and to the layout of the BSS, as well as different assumptions in the nonlinear equilibrium model of the BSS, may change the ranking of effects on sensitivity of the design. This sensitivity analysis is specific to the BSS design, and may not reflect ranking for all BSSs.

5.9 Robustness of the selected compromise

A decomposition of grid-search optima shows that trade-offs occur at the level of both selection and placement. Numerical selection refers to the fact that the selected candidate is the grid point with the lowest score among the set of points representing

the design criteria. Placement refers to the fact that the selected candidate is located at the lower part of the rise boundary. Therefore, the candidate is compromised with respect to the design domain, especially if the ceiling of the rise is lowered due to structural constraints. For example, if the configuration of the rise is lowered due to structural constraints, the design may have to be revised. In grid search, the outcome depends on the hypotheses that are entered into the model. Therefore, before such conclusions are communicated to the client, all design constraints should be verified.

Another concern is the stress margin. In our case, the margin is 6.25% of the stress threshold of 600 MPa. This should not be confused with the uncertainty in the stress margin. There are numerous sources of uncertainty, including loss of prestress, force re-distribution, and fabric-plane constraints, which are not modeled. Thus, re-ranking and re-evaluation of adjacent candidate designs should be performed prior to communicating the results of the analysis to the client.

The sensitivity check is equally applicable to the final score. Using equal weights illustrates the methodology, but leads to unexpected outcomes. For e.g., if the adjudicator is more inclined to give higher weights to deformation, the material with less deformation may win. Performing sensitivity analysis (keeping all else fixed) in an EDIS (extended decision-in-support system) framework, will inform the decision-maker if there exist a set of competitors (near) neighbors for the leading candidates. Another extension of the EDIS framework will be to apply non-dominated sorting for both the alternatives and score them, thus providing a way to visualize the dominance and eliminating the need to combine them to a single score.

6. OPTIMIZATION OUTCOME AND DESIGN INTERPRETATION

6.1 Selected compromise.

The equal-weight score selects the candidate with the least combined normalized values of cable-volume proxy and midspan deflection among the candidates passing the two screening constraints. The selected candidate is located at the rise range, the area range, and at an intermediate prestress value. A reasonable interpretation of the design is that a higher rise provides adequate stiffness to reduce the amount of cable needed, and a lower value of prestress indicates that the force is not fully utilized, thus providing a smaller amount of deflection. It is worth noting that the candidate is located at two boundary ranges, indicating that the influence of the range, weight, and constraint select the design. If architecturally higher rises are not available, or a smaller cable area is constrained by a product requirement, the design may need to change.

Table 3. Optimization outcome

Indicator	Selected compromise	Minimum-deflection candidate
Rise-to-span ratio	0.1500	0.1500
Cable area (mm^2)	400	1200
Prestress (kN)	225	300
Midspan deflection (mm)	10.0457	5.6926
Nominal stress (MPa)	562.5000	250.0000
Cable-volume proxy (m^3)	0.0053	0.0158
Feasible grid candidates	858 of 891	858 of 891

6.2 Why the score is not a universal optimum.

The score balances the normalized volume proxy and normalized deflection. Equal score balancing does not imply equal economy and safety importance. For example, a project aiming to achieve a particular roof depth may need to compromise rise; a project aiming to achieve serviceability may need to give more importance to deflection. It depends on the project constraints. A fixed cable product may consider area as a continuous variable. Likewise, 40 mm deflection and 600 MPa stress may be screening limits. A design must consider appropriate service limit state, code, combination of loads, resistance factors, and other design constraints. Hence, the cases presented are instances of a reproducible solution to the optimization problem.

6.3 Pareto interpretation.

In the absence of a formal evaluation, the cases presented represent the Pareto solution. Candidates represented in the case study may be dominated if a different design may satisfy the criteria with a strictly better volume proxy, deflection, or both. Dominated cases may or may not be represented in the case study. It is useful to represent dominated cases, as a design may be evaluated based on project priorities. For example, the next case study may report an explicit Pareto solution and include additional constraints such as mass, carbon footprint, anchorage demand, and life cycle cost.

6.4 Design principle 1—select geometry before fine-tuning prestress.

Among all variables in the model, rise is the most dominant variable in predicting deflection. An early design decision could be to establish an admissible range of the rise-to-span ratio to account for clearances, structural and operational depths and force paths, and then to optimize prestress to the greatest extent possible. Balancing all of these factors, including rise, in the overall design aligns with the existing optimization literature in BSS (Xue & Liu, 2008, 2009). It is important to manage rise as an architectural and a structural variable, while assessing it in conjunction with other variables such as cable length, rigidity and limits of the system.

6.5 Design principle 2—size the cable using rigidity and resistance together.

Reduction of deflection should not be the only consideration in determining cable cross-sectional area. The design should consider other constraints, including the mass of the cable, the service-force increments, the restraining system, and the CFRP modulus and strength. Transverse dimensions of CFRP present challenges for anchorage. The constraints should be considered in combination. The objective of this analysis is to assess the order of magnitude of the system. The values of 600 MPa used in the analysis should not be interpreted to mean that design values should be checked against 600 MPa. Rather, values relevant to the product should be considered.

6.6 Design principle 3—treat prestress as a bounded design variable.

Prestressing should ensure that loads are accommodated within the limits of member and connection stresses and services constraints. The use of prestressing may reduce (or eliminate) some load induced displacement; however, it may increase the levels of cable and strut forces, support reactions and anchorages. The small displacements predicted for this model reflect the modeling approach and assumptions, and do not reflect a lack of significance of prestressing on the overall model behavior. Equilibrium analysis should be performed for the various states of the cable including initial strain and final service and ultimate state to evaluate the response of the structure.

6.7 Design principle 4—separate screening from verification.

A simplified model is adequate for identifying unacceptable load cases and broad patterns of behavior. Such a simplification does not replace a detailed analysis considering geometric nonlinearity, imperfections and other design limits. For the vast majority of cases, it is better to begin with a simplified model and enhance it with respect to a smaller number of cases representing a broad spectrum of behavior. These cases should represent the most likely limits of behavior for the structure and be examined with respect to appropriate benchmarks.

6.8 Design principle 5—report the margin and the objective.

For each recommended candidate, the constraints satisfied, margins to constraints, objective weights, parameter limits, and the model scope should be included. In this case, the nominal stress of 562.5 MPa is less than the scenario limit of 600 MPa, providing a 37.5 MPa margin according to the screening rule. That margin is not a code-based safety factor, but reporting it allows the reader to evaluate the impact of changing the assumptions and criteria for the design. If the project requirement for allowable stress is set lower, this candidate may not have been allowable, and the analysis would have to be restarted.

7. DISCUSSION

7.1 Relationship to prior BSS optimization.

The present findings confirm BSS is affected by geometry and prestress and confirm that the optimum position is closer to the center of the span. The present results do not corroborate the numerical results of Xue and Liu (2009) and Xue and Liu (2008). The authors of the referenced studies performed ANSYS-based analyses and tested physically (or virtually) a structure of their

design, and considered effects of geometry and loading. The present model consisted of a simplified system of a beam and springs. Conceptually, the present work provides a systematic approach to organize a design problem. Prior work by Xue and Liu provides evidence of checking serviceability and ultimate limit state of the structure. The present optimum can not be compared to a physical system without equivalent consideration of multiple design variables such as span, section of beam, arrangements of cables, types of load, and properties of the material, and the objective function.

7.2 Why the rise effect is strong here.

Rise effect is based on the equivalent spring model. Stiffness of the vertical direction due to tension in cable is given by $(EA/l_c)c_v^2$. If the vertical distance is greater than the horizontal distance, then c_v increases, and accordingly a larger portion of the axial stiffness of the element contributes to the vertical direction. This is evident in the model; however, the amount of rise effect is dependent on the chosen equivalent model. In an exact BSS (building structure system), force is distributed across the elements in a manner which may not be accounted for by the spring model. Hence, the rise effect should be investigated on an explicit model which accounts for shape of the cables and the arrangement of struts.

7.3 Why prestress appears weak in the present deflection results.

In an equivalent spring model, the term T_0/l_c contributes to the model stiffness in the prestressed direction, while the material term is determined by E_cA_c. Considering the value of the given constants, the stiffness contributed by the material is greater. Therefore, in a model where a prestressing force of 100 kN in tension is increased to 300 kN, there is not much change in the equivalent stiffness. As such, the behavior of the model is not changing much. It can be stated that, in a geometrically non-linear model, the adjustment of prestressing forces may bring significant change in equilibrium configuration of the model. Hence the results obtained should be analyzed in conjunction with other models, which show the mechanism of action of the model.

7.4 CFRP substitution and equivalence.

The equivalence conditions that define the basis of comparison between steel and CFRP systems enable different analyses. For example, the current study determines the effects of area and prestress in CFRP systems and does not attempt to position CFRP systems against steel systems. Published reports on CFRP cable systems identify benefits and constraints. Those reports warn that cable form and anchorage should be taken into account (Liu et al., 2015; Yang et al., 2020). Other reports also emphasize that the primary barriers that limit the use of CFRP cable systems are integration of material properties and performance of the structural system and the nature of the system's connections (Meier, 2012). Thus, the scope of future comparative studies should address the aforementioned limitations and consider physical dimensions, mass and anchorage of CFRP cable systems.

7.5 Serviceability versus resistance.

The model has been developed to evaluate serviceability and to estimate the order of magnitude of the materials utilized. It is not developed to evaluate the ultimate strength or to assess cable failure due to loss of prestressing, non-linearity or buckling. Therefore, the phrase "load-bearing capacity" should be interpreted in the sense of a research gap and not in the sense of an evaluation of the real capacity of the system. Other research has investigated serviceability and evaluation of the ultimate capacity of BSSs (Xue & Liu, 2009). These research stages are necessary for the development of a system that allows the consideration of BSSs in engineering practice.

7.6 Model credibility and reproducibility.

Most aspects needed to reproduce the model are described and include the finite element mesh, material and boundary conditions, equations, constraints, and a definition of the score. The authors provide a CSV file with 891 candidates to allow the reader to examine the output and adjust the objective function. The authors indicate that standard Python and NumPy libraries together with SciPy, which provides numerical routines and tools for scientific computing, can be used to implement the numerical analysis (Virtanen et al., 2020). While the authors indicate that their model and analysis can be reproduced, they do not validate the physical model. Even if the analysis can be repeated, it may be a poor representation of the physical system. Credibility in this case, relies on both the analysis and the representation of the physical system.

7.7 Practical interpretation.

In the case of the geometry and loading considered in this paper, larger cables, all other things being equal, would lead to less deflection, and for a given cable area, too much prestress would negatively impact the model. The deflection model is typically a good starting point in the design of a cable stay structure. Prior to specifying dimensions, a designer should perform a full structural analysis, consider other modes of failure (e.g. ultimate and serviceability limits, cable rupture, anchor pull-out, etc.) and check the model against published practice, or benchmarking data and/or a dedicated physical test.

8. DESIGN RECOMMENDATIONS

The first recommendation is to determine the design objective prior to use of an optimizer. This objective may specify the minimum level of cable deflection, the minimum cable count, the minimum mass, the minimum cost, or a compromise of several of these. In the case of multiple criteria, recommendation was made to include a justification for the weights or to report the Pareto optimal solution. The current case represented an equal weighting for all criteria as a baseline. This is to be considered as an example, and not to be generally applied.

The second recommendation is to apply project limits to the design criteria. These may include limitations of deflection and/or stress, or permitted prestress. Stability conditions may also be included. These limits should be determined prior to optimization. In the paper, the two deliberately simple limits are examples to illustrate the procedure. These examples by no means define acceptable limits for practice. In general, the objective of a design should be to fully integrate the limitations of the project and available standards.

The third recommendation is to consider together changes in prestress, rise and cable area. One change at a time may be done to illustrate a concept, however, a combination of multiple variables may be better assessed using a design of experiments. The author has generally taken this approach. In the case of a design of experiments, a larger number of variables may be assessed by limiting the scope of the experiment.

The fourth suggestion is to implement a "product-conscious" material model. The modulus and tensile strength of CFRP should be defined by the properties of the respective cable/tendon product. Statistical values should be determined based on the product, design values should be ex fault. Cable area should be compatible with the anchorage and stressing systems. The global axial model should not be used to assess the safety of anchor zones. Transverse and interlaminar behaviors of CFRP are distinct from longitudinal behaviors. Therefore, for such assessment, other coupled constitutive models should be used in conjunction with the global axial model. The optimization workflow should disregard candidate zones that cannot be anchored or validated to carry the required force.

The fifth recommendation is to differentiate between the initial state (or design) and service loading cases. The model should be formulated to consider the application of the initial prestressing. It should then be analyzed to check if the system is, in equilibrium and how much it deflects after prestressing, and after other service loading cases. The representation of the stressing process should be incorporated in the model if the variation in compliance of cable anchorage materially affects the force state.

The sixth recommendation is to keep a clear audit trail. A trace should be kept of the code version and all input parameters (including mesh definition, boundary conditions, objective weights and constraints), as well as solver settings. All potential design solutions should be documented. The CSV enclosed with this report documents a design case. An exhaustive design space exploration requires that the starting point and the random number generator seed be saved. The configuration of the optimization algorithm should also be recorded. The design solution should be checked to ensure that the result is not spurious (i.e. a result of poor choice of boundary conditions or numerical approximation).

The seventh recommendation is to use an evidence hierarchy. Simple benchmark problems and mesh refinement should be used to confirm the code is working correctly. The structural response of the code should be compared against an independent industry benchmark or a published structural test of interest. If these tests indicate the code is performing as desired, and the structural components being designed demonstrate the desired response, full non-linear analysis of the structure should be performed, including an analysis of the cables and struts of the structure. If structural elements are to be designed using unverified connection behavior, they should be tested as part of the component or sub-assembly level evaluation. The complete structure, and each of the load combinations that affect it, should be tested to prove the structure is safe.

9. FUTURE WORK

The next stage of modeling should replace equivalent springs with cable and strut elements, and represent cable elements as having only tension. Cable elements should be constructed using either form-finding or the principles of equilibrium. Depending on the requirements of the analysis, the effects of geometric non-linearity and cable tension should be included. These enhancements should be added so that the design case study can explore the effect of the system on the tangent modulus. This includes the effects of system stiffness and initial load.

An additional enhancement to the model would include consideration of stability. This would include nonlinear buckling analysis or other types of analysis that consider equilibrium or constrain the slack condition of cable elements. Stability limits would also need to be incorporated into the analysis of struts and beams. The effects of geometric imperfections and material resistance would also need to be considered. The final model and the constraints incorporated into it would govern the possible scope of the conclusions that can be drawn from the case study, including potential load factors and stability limits.

A third extension would be to use the actual cable properties of CFRP and incorporate the CFRP anchorage behavior in the model. The global design parameters would be linked to the actual cable size, modulus, strength, density and a qualified anchorage system. Anchorage compliance or seating loss would be modeled using load-slip elements. CFRP anchorage has been researched and published and would provide input for component tests to be utilized. Local failure modes would need to be assessed, as with any new BSS design (Han et al., 2017; Wang et al., 2018; Wang et al., 2024; Züst et al., 2022; Puigvert et al., 2014).

A fourth extension would be experimental or benchmark validation. As a basis for validation, the design model would need to be loaded to a defined displacement and a load-deflection response would be expected. The model would predict cable forces, beam deflections, and possibly strut forces. Strut failure would be expected based on the load level. Validation would be performed against a load/deflection response. It should be noted that in the case of published data insufficient to perform a complete validation, this would be documented and an equivalent test/validation program would be performed. A good comparison of the overall model response would not be acceptable.

The last extension described multi-criteria design. Here, I consider multi-objective design. In addition to cable-volume proxy and deflection, other project-related design criteria could incorporate mass of cables and beams, embodied carbon, fabrication, anchorage, life-cycle cost, and maintenance. The aforementioned design criteria can be embedded in a design model. With material and cost data, other design constraints can be defined. For example, the Pareto set can be defined for designing a serviceable structure. Variable-based and screening designs can determine which design criteria influence the serviceability the most.

A final aspect of design under uncertainty is robustness. In practice, there are several conditions that are beyond the designer's control. These include, but are not limited to, variations in material properties, fabrication and installation tolerances, change in temperature, support settlement, and loads that are not symmetric or balanced. It is not sufficient to assess a candidate design solely by its performance at a design point. Samples of uncertain variables should be drawn from their specified ranges. The candidate designs should be compared by the possible worst-case scenario responses. It should be noted that in determining the Design under Uncertainty (DuU) aspect, the constraints on the design are as important as the objective function, and these should be clearly distinguished. There are other areas that need to be documented and published, and this will be provided in the next paper. The methods for Continuous Design Optimization" are as varied as the inventive methods for Discrete Design Optimization. Storn and Price (1997) provide an excellent source for design methods that utilize differential evolution. Other references are available in scientific and mathematics-oriented Python modules, e.g., (Virtanen et al., 2020). If the number of variables is relatively small, then a grid search would be most appropriate. If the parameters are of an appreciable number and/or vary in a complex manner, then a search utilizing methods of design and optimization would be more appropriate. In all cases, design methods should be preceded by verification of the design and modeling elements. Thus, if a design has been shown to exhibit desired and stable performance, then it is appropriate to allow the methods of Continuous Design Optimization to operate.

10. CONCLUSIONS

We developed an efficient computational workflow to quickly assess rise-to-span ratios, cable areas, and prestressing levels in a cable-based beam-and-string system. The workflow relies on a 48-element model of Euler-Bernoulli beams and two equivalent

vertical tangent springs to capture the stiffness of inclined cable-struts. The complete workflow assesses 891 designs and captures information such as the midspan deflection, the level of prestress and stiffness of the springs, approximate cable length, and a proxy for cable volume.

As expected, increasing the rise-to-span ratio reduces midspan deflection. An increase in cable area also reduced midspan deflection and nominal prestress, however the proxy for cable volume increased. Increasing the level of prestress from 100 kN to 300 kN resulted in a slight decrease in deflection and a significant increase in the nominal stress of the cable. These results show that changes in midspan deflection of the model, for the rise-to-span and cable area ratios, depend on the constraints and variables of the model, and should be interpreted in the context of the model equations.

Of the 891 designs modeled, 858 met the span/300 deflection criterion, with a nominal-stress of 600 MPa. The design with the lowest score in an equal-weighting objective, had a rise-to-span ratio of 0.15, a cable area of 400 mm2 and a prestress of 225 kN. This design had a midspan deflection of 10.05 mm and a nominal-cable-stress of 562.5 MPa. The design with the lowest deflection was modeled with a larger cable area and prestress. The design is a balance of the objectives for midspan deflection and the proxy for the material.

The results show five design guidelines: consider the effect of rise in your design; select cable area based on both rigidity and strength; limit prestress to levels allowed by the connections and the structure; provide an objective function to be optimized, along with relevant constraints; and perform reduced order analyses to provide evidence to support design decisions. The results shown are from the Python model and are not from ANSYS or from physical experiments. This work computes only linear forces and does not consider the effect of geometry on the nonlinearity of the system. Future work should consider nonlinearity, equilibrium, stability, and equilibrium" path of the system and integrate more realistic and representative constitutive models of the CFRP. Prior to accepting design-level conclusions, this work should be supported by verification and/or validation using physical experiments or comparison to other relevant work.

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