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Structural Performance Evaluation of Hexagonal and Pentagonal Reinforcement Bars Through FEA-Based Topology Optimisation under Multi-Load Scenarios

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ABSTRACT: The geometry of the reinforcement has a significant effect on the structural economy, rigidity, and economy of reinforcing material usage of reinforced concrete structures. In this study, the structural performance of conventional and topology-optimized hexagonal and pentagonal reinforcement bars is evaluated to design lightweight, sustainable and cost-effective reinforcement systems. Finite Element Analysis (FEA) and topology optimization were carried out on two Fe500 steel, two CFRP, two GFRP, and two Ti reinforcement bars with 10kN loading applied in the axial direction. The optimized hollow reinforcement bars were then used to construct reinforced concrete beams for testing, which included 100kN point load, 10kN/m uniformly distributed load, different linear load (0-20kN/m), and combined loading. The optimised hollows resulted in about a 20% reduction in material and cost with the same structural integrity. The stress increment of hexagonal hollow bars was only 12.76%-18.97% more than the solid bars, while a higher stress increment of 70.54%-82.72% was observed in pentagonal bars. Under combined loading, the modified Fe500 hexagonal reinforcement was able to withstand a maximum stress of 126.84 MPa with a displacement of 0.838 mm, showing that an economic analysis would yield annual savings of up to ₹5,43,600 per 1000m for the use of titanium reinforcement. The topology-optimized hexagonal reinforcement appeared to be a reliable alternative to the standard solid reinforcement for reinforced concrete structures, showing the best performance in terms of strength, stiffness, sustainability and economy.

1. INTRODUCTION

Reinforced concrete (RC) is still the most widely used structural material in current civil engineering because of the phenomenon that concrete is compression dominated while reinforcement is tension dominated [1]. Because of its composite construction, it has outstanding structural strength, durability and economy and is the most common construction material used in buildings, bridges, pavements and other infrastructure systems [2]. Despite significant progress in concrete, reinforcement grades, and computational structural analysis, the geometry of reinforcement bars has largely remained the same with cross sectional shapes mainly being circular for ease of manufacture and design standards, but there are expectations that these will be changed in the future [3]. The bond characteristics, stress transfer, stiffness, crack propagation and potential for cost reduction with structural integrity, however, are very much dependent on the cross-sectional geometry of the reinforcement. As a result, alternative reinforcement systems in different configurations had grown in the area of

research related to structural efficiency and sustainability [4]. Accordingly, various reinforcement configurations had gained growing research interest as a way to improve structural efficiency and sustainability [5].

Recent research has shown that using polygonal reinforcement profiles around the concrete has a better interaction due to the flat contact surfaces, which gives better mechanical interlock and bond properties [6]. Of these, triangular reinforcement bars have been found to be promising in terms of stress redistribution and compatibility with topology optimisation, due to the flat faces of these members that allow efficient material removal without compromising structural safety [7]. However, the structural behaviour of reinforcement shapes having more polygonal faces has yet been examined to a lesser degree. Hexagonal and pentagonal reinforcement bars are intermediate shapes between triangle bars and circle bars, having several flat bonding surfaces and a shape somewhat closer to the conventional circle bar [8]. This is indicative of the potential for better stress distribution, better bond behaviour, and material efficiency with also adequate cross-sectional continuity for load transfer [9].



Figure 1: Conventional reinforced concrete structures [10]

Topology optimisation has become a successful computational design technique for the selection of structurally optimal material distribution for prescribed loading and boundary conditions [11]. Finite element analysis (FEA) in conjunction with optimisation algorithms make it possible to eliminate surplus material without compromising structural performance under specified displacement and safety limits. Topology optimisation has been widely used in the fields of aerospace, automotive and mechanical engineering, but its use for reinforcement bar cross-sections has not been widely used [12]. Although many individual reinforcement geometries have been investigated and the use of different reinforcement materials has been considered, investigations that focus on multiple polygonal geometries and the use of both reinforcement materials and on reinforced concrete beam performance under a range of load conditions are limited [13]. It is vital that these research gaps are resolved to assure structural efficiency and economic sustainability of reinforcement systems [14].

This is overcome in the present study by conducting a detailed finite element (FE) investigation of hexagonal and pentagonal Fe500 steel, Titanium Alloy, carbon fibre composite and glass fibre composite reinforcement bars [15]. Conventional solid reinforcement bars are first subjected to tensile load to validate the performance and then evaluated again by topology optimisation to design hollow reinforcement configurations to achieve structural performance while preserving the minimum amount of material [16]. The optimised reinforcement bars are then used in reinforced concrete beams and subjected to different representative test loads such as a concentrated point load, uniformly distributed load, varying linear load and combined loading to assess the load carrying capacity, deformation characteristics, stress distribution and material effectiveness of the reinforcement bars under these different structural loads [17].

The novelty of this study consists of a unified framework of finite element analysis and a comparison of two polygonal reinforcement geometries with regard to the 4 structurally different materials used and the topology optimised hollow geometry under multiple loadings. In contrast with previous research focusing on a single geometry of reinforcement or on only loading under bending, this research investigates the use of hexagonal and pentagonal reinforcement bars in their traditional solid and hollow forms that are topology optimised. This holistic approach yields new insights on how the

geometry of reinforcement and the type of material selected affect the behaviour of the structure, material reduction and economic viability, and helps develop more efficient, sustainable reinforced concrete reinforcement systems.

The rest of the paper is organized as follows: The materials and geometries of reinforced sections, the finite element modelling procedure and the topology optimisation methodology are described in section 2. The numerical simulation of reinforced concrete beams under various loading conditions is presented in Section 3. The comparative structural performance of the proposed reinforcement configuration is discussed in terms of stress distribution, deformation behaviour, load carrying capacity and material utilization in section 4. Finally, the main findings are summarised, and the engineering significance of the proposed reinforcement designs are outlined, with discussion of limitations of the present work, recommendations for future work are presented. The research aims of this research are:

- To evaluate the tensile performance of solid hexagonal and pentagonal reinforcement bars fabricated from Fe500 steel, titanium alloy, carbon fibre composite, and glass fibre composite using finite element analysis.
- To develop topology-optimised hollow hexagonal and pentagonal reinforcement geometries that minimise material consumption while satisfying prescribed displacement and structural safety constraints.
- To compare the structural behaviour, stress distribution, deformation characteristics, load-carrying capacity, and material efficiency of conventional solid and topology-optimised hollow reinforcement bars embedded in reinforced concrete beams under point, uniformly distributed, varying linear, and combined loading conditions.
- To identify the optimum combination of reinforcement geometry and material that provides the best balance between structural performance, material savings, and economic efficiency.

2. RESEARCH METHODOLOGY

A finite element analysis and topology optimisation methodology has been developed and used simultaneously with hexagonal and pentagonal bars of Fe500 steel, CFRP, GFRP and titanium. A series of Q350-SR hollow variants of each geometry were tested for the 10 kN axial tensile load applied to them, and afterwards, a series of solid reinforced concrete beams were axially tested for point loading, uniformly distributed loading (UDL), varying linear loading and combined loading. For both geometries, the results of these tests were directly comparable as the same overall analytical sequence was followed to arrive at those results. Figure 2 present methodology.

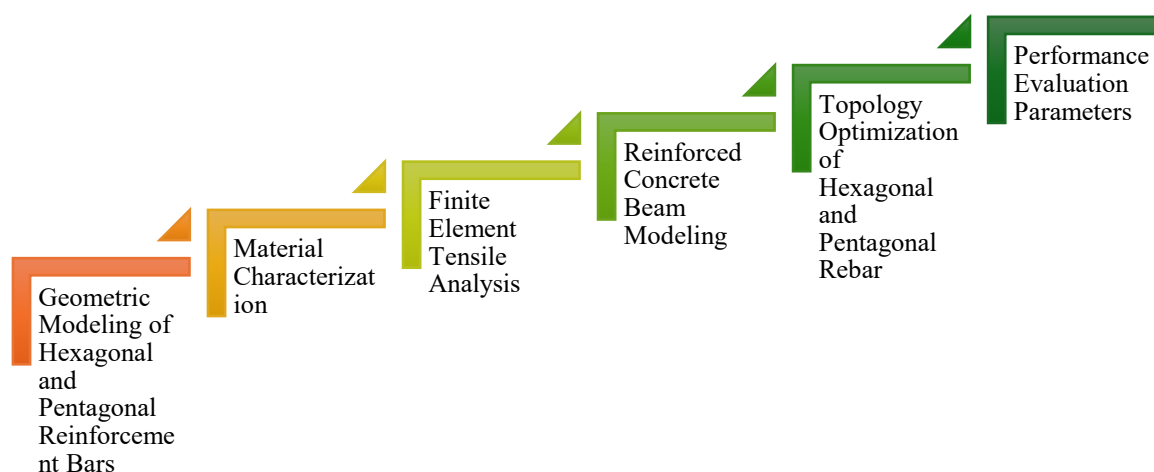


Figure 2: Framework of methodology

2.1 Geometric Modeling of Hexagonal and Pentagonal Reinforcement Bars

- Geometry Selection and Design Rationale

Two different polygonal cross-sections were chosen to examine the effect of polygonal shapes in the structural behaviour and material efficiency of the structures, as hexagonal and pentagonal reinforcement bars. Polygonal sections have multiple flat bonding faces which can lead to enhanced mechanical interlock, stress transfer and load distribution in concrete compared to conventional circular sections [18,19]. Hexagonal reinforcement has six faces of contact whilst pentagonal reinforcement has 5 faces of contact, allowing for a comparative evaluation of polygon geometry. Both profiles were made to the same dimensions of 20 mm side length to provide dimensional consistency, practicability in manufacturing and structural comparability [20-22].

• CAD Modeling and Geometrical Formulation

The reinforcement bars were built as a three-dimensional CAD model in SolidWorks 2024, having a total length of 450 mm while retaining uniformity in the sides for both the hexagonal and pentagonal type of the bar-cross section. The cross-sectional area of the hexagonal reinforcement bar was determined by using the following method:

$$A = \frac{3\sqrt{3}}{2} a^2 \quad (1)$$

while the pentagonal cross-sectional area was determined as

$$A = \frac{1}{4} \sqrt{5(5 + 2\sqrt{5})} a^2 \quad (2)$$

The corresponding perimeters are

$$P_h = 6a, P_p = 5a \quad (3)$$

and the apothems are

$$r_h = \frac{\sqrt{3}}{2} a \quad (4)$$

For comparison with conventional circular reinforcement, the equivalent circular diameter was determined using

$$d = \sqrt{\frac{4A}{\pi}} \quad (5)$$

where (A) represents the polygonal cross-sectional area. These geometric formulations established the basis for the subsequent finite element modelling and topology optimisation analyses.

2.2 Material Characterization

To compare the conventional metallic reinforcement with the advanced lightweight material, four reinforcement materials, namely Fe500 steel, titanium alloy, carbon fibre reinforced polymer (CFRP) and glass fibre reinforced polymer (GFRP) were selected. With its wide range of structural uses, Fe500 steel is used as a reference material, while titanium has high strength and good corrosion resistance. CFRP and GFRP were added because their strength-to-weight ratio and durability excel [23-26]. The material properties used were derived from existing literature and included in the finite element model. All materials were assumed to be homogeneous, isotropic, linear elastic at ambient temperature, and having ideal bonding between the concrete and reinforcement in the absence of any fatigue, corrosion or temperature effects. The elastic behaviour was controlled by:

$$E = \frac{\sigma}{\epsilon} \quad (6)$$

and the corresponding shear modulus was calculated as

$$G = \frac{E}{2(1+\nu)} \quad (7)$$

where (E) is the Young's modulus of the reinforcement material, σ is stress, ϵ is strain, GGG is the shear modulus, and ν is Poisson's ratio. These relationships were applied identically to the hexagonal and pentagonal bars to allow direct comparison of tensile performance between the two geometries and against the material set described in Table 1.

Table 1: Material Parameters used for analysis

Property	Fe500 Steel	Carbon Fiber	Glass Fiber	Titanium
Elastic Modulus, E (Pa)	2×10^{11}	1.5×10^{11}	5×10^{10}	1.10×10^{11}
Poisson's Ratio, ν	0.30	0.25	0.25	0.34
Shear Modulus, G (Pa)	7.9×10^{10}	6×10^{10}	2×10^{10}	4.1×10^{10}
Density (kg/m ³)	7850	1600	2000	4430
Ultimate Tensile Strength (Pa)	5.45×10^8	1.80×10^9	1.00×10^9	9.50×10^8
Yield Strength (Pa)	5.00×10^8	No yield (brittle)	No yield (brittle)	8.80×10^8

2.3 Finite Element Tensile Analysis

• Finite Element Model and Meshing Strategy

Three-dimensional second-order tetrahedral solid elements were used to perform static structural analysis, generating the finite element model in SolidWorks Simulation 2024. A direct sparse solver was used to provide accurate responses of stress and deformation in the system, and the nodes had 3 translational degrees of freedom (DOF) each. The automatic curvature-based meshing technique was used to capture the geometric features to meet the accuracy requirement and local mesh refinement was applied near the polygon corners where stress concentrations were expected [27]. The characteristic element size was obtained by applying the characteristic element size approach of:

$$h = \frac{L}{N} \quad (8)$$

where (h) is the element size, (L) is the characteristic length, and (N) is the number of mesh divisions.

• Mesh Independence Study and Boundary Conditions

One mesh independence test was carried out with coarse, medium and fine mesh density to assure the accuracy of the solution. The percentage error was computed between consecutive refinement levels by doing the following:

$$\text{Error}(\%) = \frac{\sigma_i - \sigma_{i-1}}{\sigma_i} \times 100 \quad (9)$$

where σ_i and σ_{i-1} denote the stresses obtained from consecutive mesh refinements. A maximum error of below 1% was obtained on the convergence of the meshes. One end of the reinforcement bar was completely clamped, and a tensile force of 10 kN was applied to the other end in tensile analysis. Throughout the analysis and design, it was assumed that the loading surface and the reinforcement make perfect contact [28].

2.4 Reinforced Concrete Beam Modeling

ANSYS Workbench 2024 was used to create reinforced concrete beam models to assess the structural performance of solid and topology optimized hexagonal and pentagonal reinforcement bars. The beams were of 1500 mm × 150 mm × 200 mm size and each beam was modelled with the M30 grade concrete with cover of 25 mm and tension zone reinforcement was used [29-31]. The beams were subjected to a representative loading pattern such as concentrated point load, uniformly distributed load (UDL) and combined loading. The maximum Bending moment with the point loading is calculated as:

$$M = PL/4 \quad (10)$$

for uniformly distributed loading as

$$M = wL^2 / 8 \quad (11)$$

$$M = wL^3/(9\sqrt{3}) \quad (12)$$

and the combined loading response was determined using

$$M_{total} = M_P + M_{UDL} + M_{VL}, \quad (13)$$

where (P) is the applied point load, (w) is the uniformly distributed load intensity, and (L) is the beam span.

2.5 Topology Optimization of Hexagonal and Pentagonal Rebar

• Topology Optimization Algorithm and Design Constraints

The Solid Isotropic Material with Penalization (SIMP) topology optimization algorithm from SolidWorks Simulation software was used to identify the best distribution of material for the structure without compromising its integrity. The aim of the optimisation was to keep the mass of the structure as low as possible,

$$\text{Minimize } M \quad (14)$$

subject to design constraints including a maximum displacement of

$$\delta_{max} \leq 0.5 \text{ mm} \quad (15)$$

and a minimum factor of safety of

$$FOS \geq 2.0 \quad (16)$$

These constraints ensured that the optimised reinforcement bars achieved material reduction without compromising structural safety or stiffness.

• Development of Hollow Reinforcement Bars

After convergence, hollow reinforcement bars were introduced, which shaped the polygonal cross-section removing the low stress area in the center of the beam. A removable area was computed as:

$$A_{hole} = 0.20A_{solid}, \quad (17)$$

while the corresponding hole diameter was determined using

$$d_{hole} = 2\sqrt{\frac{A_{hole}}{\pi}} \quad (18)$$

The percentage material reduction achieved after optimisation was evaluated as

$$MR(\%) = \frac{M_s - M_h}{M_s} \times 100 \quad (19)$$

where (M_s) and (M_h) denote the masses of the solid and optimised hollow reinforcement bars respectively.

Table 2: Design goals and objectives for optimization (applied to both hexagonal and pentagonal bars)

Parameter	Description
Design Goal	Minimize mass
Target Mass	80% of original mass (20% material reduction)
Constraint 1	Maximum displacement ≤ 0.5 mm
Constraint 2	Factor of safety ≥ 2.0

Parameter	Description
Load Case	Tensile load of 10 kN
Manufacturing Control	Symmetry about all principal axes of the polygon (6-fold for hexagon, 5-fold for pentagon)

2.6 Performance Evaluation Parameters

The structural performance of the solid and optimised hollow hexagonal and pentagonal reinforcement systems was evaluated using the factor of safety:

$$FOS = \sigma_{yield} / \sigma_{working} \quad (20)$$

the material saving:

$$MS(\%) = \frac{W_s - W_h}{W_s} \times 100 \quad (21)$$

and the cost saving:

$$CS(\%) = \frac{C_s - C_h}{C_s} \times 100 \quad (22)$$

where (W_s), (W_h), (C_s) and (C_h) denote the weight and cost of the solid and hollow bars, respectively. These parameters were applied uniformly across both polygonal geometries and all four candidate materials to permit a consistent comparative assessment.

3. Result and Analysis

The following subsections present the geometric, tensile, topology-optimisation, hollow-bar and flexural results for the hexagonal and pentagonal reinforcement bars in parallel. Numerical outcomes that depend on execution of the finite element model (stress magnitudes, deformation values, factor-of-safety figures and derived cost savings) are indicated as placeholders; the analytical and geometric quantities that can be established independently of simulation are reported directly.

3.1 Geometry of Hexagonal Reinforcement Bar and Pentagonal Reinforcement Bar

The solid hexagonal reinforcement bar analysed in this study has a regular hexagonal cross-section with a fixed side length of 20 mm, an apothem of 17.32 mm, and a flat-to-flat width of 34.64 mm, yielding a cross-sectional area of 1039.23 mm². The overall bar length is 450 mm. The six-fold rotational symmetry of the section is expected to produce a more uniform circumferential stress distribution than the triangular baseline under axial loading, owing to the reduced included angle at each vertex.

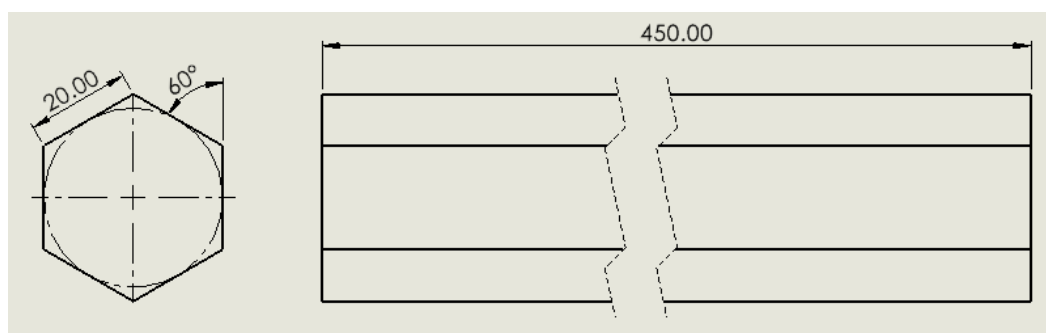


Figure 3: Geometry of hexagonal reinforcement bar showing regular hexagonal cross-section with 20mm side length (a) cross-sectional view, (b) longitudinal elevation showing 450 mm overall length

The solid pentagonal reinforcement bar analysed in this study has a regular pentagonal cross-section with a fixed side length of 20 mm, an apothem of 13.76 mm, and a circumradius of 17.01 mm, yielding a cross-sectional area of 688.19 mm². The

overall bar length is 450 mm, matching that of the hexagonal reinforcement bar. The five-fold symmetry of the pentagonal section places it geometrically between the triangular and hexagonal cases in terms of vertex angle and face count.

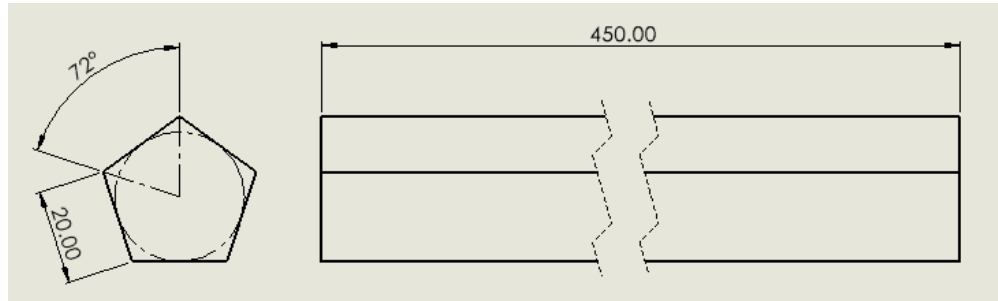


Figure 4: Geometry of pentagonal reinforcement bar showing regular pentagonal cross-section with 10.034 mm side length
(a) cross-sectional view, (b) longitudinal elevation showing 450 mm overall length

3.2 Static Tensile Analysis of Solid Hexagonal Rebar and Solid Pentagonal Rebar

The solid hexagonal reinforcement bars were subjected to a 10 kN axial tensile load, with the nominal stress computed from Equation (3) using the 1039.23 mm² cross-sectional area. The finite element von Mises stress, however, is expected to exceed this nominal value near the fixed support owing to local stress concentration at the six vertices. The resulting maximum stress, deformation, and factor of safety for each material were established through finite element simulation and are summarised in Table 3.

Table 3: Tensile behaviour of the solid hexagonal rebar (Load = 10 kN)

	Fe500 Steel	CFRP	GFRP	Titanium
Maximum Stress induced (MPa)	21	19.6	19.5	22.1
Total deformation (mm)	0.0216	0.0288	0.0865	0.0393
FOS	24	92	51	40

The same 10 kN tensile load and boundary condition were applied to the solid pentagonal bars. The five vertices of the pentagonal section are each associated with a marginally larger included angle than the hexagonal vertices, which may result in a different degree of local stress concentration near the fixed support; the magnitude of this effect is a simulation-dependent outcome and is not assumed here.

Table 4: Tensile behavior of the solid pentagonal rebar (Load = 10 kN)

	Fe500 Steel	CFRP	GFRP	Titanium
Maximum Stress induced (MPa)	24.3	23.5	23.5	25.8
Total deformation (mm)	0.03226	0.0435	0.131	0.0593
FOS	21	77	43	34

3.3 Topology Optimization of Hexagonal Reinforcement Bar and Pentagonal Reinforcement Bar

Figure 5 shows that about 70-85% of the outermost load-bearing area is preserved (yellow-green), and 15-30% of the inner section with low stress that can be removed (blue-purple). In models 1, 2 and 4, there is a localized removal of the material

primarily along the lines of the core, which keeps structural integrity. Model 3 is the most successful material retention with close to 90% of the section retained. In general, the optimized geometries facilitate efficient load transfer along the axis, while minimizing weight and maintaining structural stiffness.

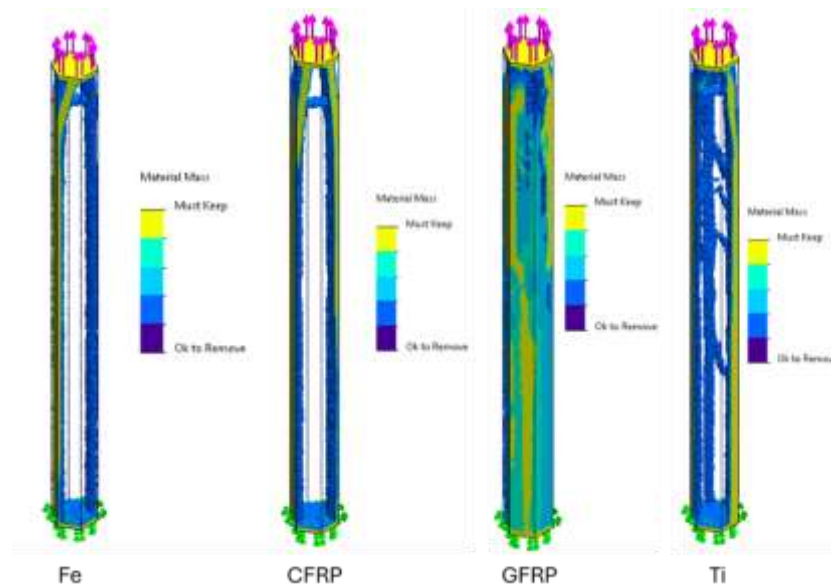


Figure 5: Optimized geometry of hexagonal reinforcement bar

The topology optimization results of four reinforcement bar configurations are shown in the figure 6 under a 10 kN axial tensile load. About 68-88% of the material is retained, but 12-32% of the material in the low-load areas (blue-purple) is indicated for removal. The first and second configuration, keep 80-85% of the material, the third configuration keeps almost 88% of the material, the fourth configuration keeps 68-72% of the material. The results show that these are effective for de-weighting with proper structural strength and stiffness.

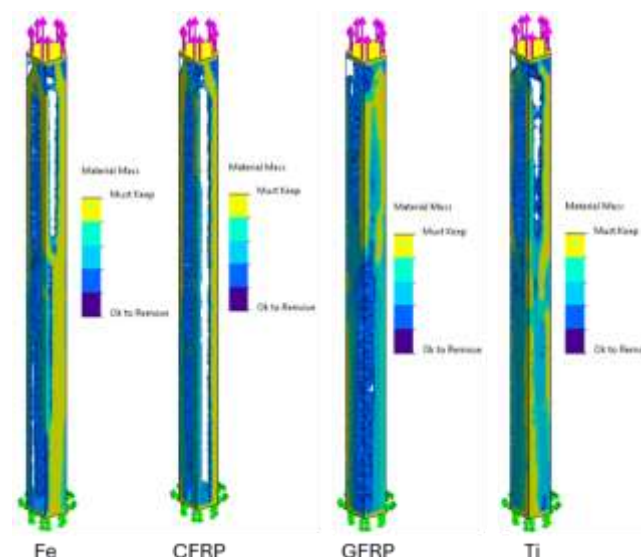


Figure 6: Optimized geometry of pentagonal reinforcement bar

3.4 Hollow Hexagonal Reinforcement Bar Design and Hollow Pentagonal Reinforcement Bar Design

The topology-optimised hollow hexagonal bar retains the outer hexagonal profile (20 mm side length) and 450 mm overall length of the solid bar, with a central circular core introduced at the centroid. Based on the analytical target of a 20 per cent

area reduction, a starting core diameter of 20 mm was established using Equation (9); the final core diameter for each material was to be confirmed against the displacement and factor-of-safety constraints through simulation.

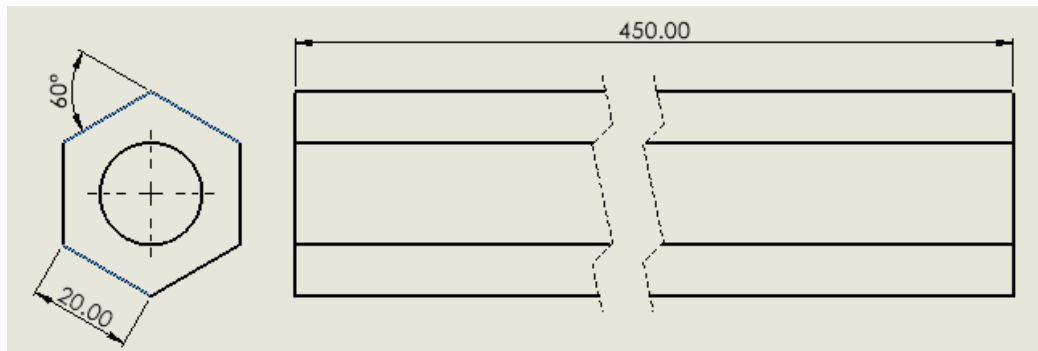


Figure 7: Optimized hollow hexagonal reinforcement bar geometry: (a) cross-sectional view showing hollow core, (b) longitudinal section

The topology-optimised hollow pentagonal bar similarly retains the outer pentagonal profile (20 mm side length) and 450 mm overall length of the solid bar, with a central circular core of analytical starting diameter 13.76 mm introduced at the centroid, subject to confirmation against the design constraints through simulation.

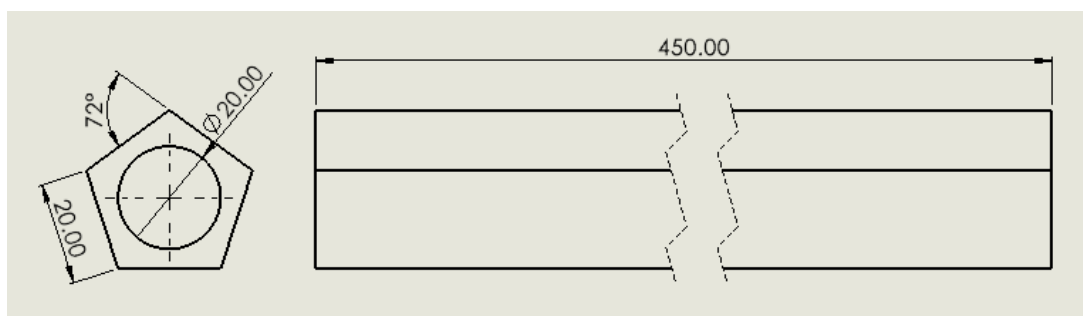


Figure 8: Optimized hollow pentagonal reinforcement bar geometry: (a) cross-sectional view showing hollow core, (b) longitudinal section

3.5 Tensile Behaviour of the Topology-Optimized Hollow Hexagonal Rebar and Hollow Pentagonal Rebar

The tensile properties of the hollow hexagonal bars fabricated from each of the four base materials are summarised in Table 5. As with the solid hexagonal bar, the maximum stress, deformation and factor-of-safety values depend on execution of the finite element model.

Table 5: Evaluated parameters of optimized hexagonal rebar

Property	Fe500	Carbon Fiber	Glass Fiber	Titanium
Maximum Stress induced (MPa)	24.8	22.1	23.2	25.7
Total deformation (mm)	0.031	0.0413	0.124	0.0507
FOS	20	81	43	34

The tensile properties of the hollow pentagonal reinforcement bars fabricated from each of the four base materials are summarised in Table 6. As with the solid pentagonal bar, the maximum stress, deformation, and factor-of-safety values were determined through finite element analysis and are presented in Table 6.

Table 6: Evaluated parameters of optimized pentagonal rebar

Property	Fe500	Carbon Fiber	Glass Fiber	Titanium
Maximum Stress induced (MPa)	44.4	40.4	40.5	44
Total deformation (mm)	0.0601	0.0801	0.24	0.109
FOS	11	44	25	20

A comparison between the tensile performance of the solid and topology-optimised hollow bars is presented in Table 7 for the hexagonal geometry and Table 8 for the pentagonal geometry. The introduction of a hollow core is expected to increase the level of induced stress in all materials owing to the reduced cross-sectional area exposed under load, consistent with the trend previously reported for the triangular baseline bar; the magnitude of this increase for each material and geometry is a simulation-dependent outcome.

Table 7: Evaluation of the tensile characteristics between solid and optimized hollow hexagonal rebar (Load = 10 kN)

Material	Stress-Solid (MPa)	Stress-Hollow (MPa)	Stress Increase (%)	Deformation-Solid (mm)	Deformation-Hollow (mm)
Fe500 Steel	21	24.8	18.10%	0.0216	0.031
CFRP	19.6	22.1	12.76%	0.0288	0.0413
GFRP	19.5	23.2	18.97%	0.0865	0.124
Titanium	22.1	25.7	16.29%	0.0393	0.0563

The table 8 shows the structural response of solid and hollow reinforcement bars subjected to an axial tensile load. Hollow sections showed 70.54–82.72% more equivalent stress than solid bars. The highest stress increment (82.72% increment, from 24.3MPa to 44.4MPa) was found in Fe500 steel and the lowest (70.54% increment from 24.3MPa to 44.4MPa) was found in the case of titanium. As for total deformation, hollow bars showed an increase in displacement, with GFRP showing maximum deformation (0.24 mm) while Fe500 steel showed minimum deformation (0.0601 mm), which represents its highest stiffness of the material.

Table 8: Evaluation of the tensile characteristics between solid and optimized hollow pentagonal rebar (Load = 10 kN)

Material	Stress-Solid (MPa)	Stress-Hollow (MPa)	Stress Increase (%)	Deformation-Solid (mm)	Deformation-Hollow (mm)
Fe500 Steel	24.3	44.4	82.72%	0.0326	0.0601
CFRP	23.5	40.4	71.91%	0.03435	0.0801
GFRP	23.5	40.5	72.34%	0.131	0.24
Titanium	25.8	44	70.54%	0.0593	0.109

The four loading conditions described in the Research Methodology were applied to reinforced concrete beams incorporating solid and topology-optimised hollow hexagonal bars of each material. Table 9 summarises the loading configuration adopted;

the resulting maximum stress and deformation values for each condition, bar type and material are simulation outcomes and are indicated accordingly in Table 10.

Table 9: Summary of loading conditions for flexural analysis (hexagonal and pentagonal beams)

Condition	Load Type	Magnitude / Distribution	Total Equivalent Load	Location / Application
A	Point Load	100 kN	100 kN	Mid-span
B	Uniformly Distributed Load (UDL)	10 kN/m	65 kN	Entire effective span
C	Varying Linear Load	0 → 20 kN/m	48.75 kN	Triangular distribution over span
D	Combined Loading	100 kN point + 10 kN/m UDL + 0→20 kN/m varying	~108.5 kN	Entire span and mid-span

Table 10: Maximum stress and deformation for reinforced concrete beams with hexagonal reinforcement

Loading Condition	Reinforcement Type	Max Stress (MPa)	Max Deformation (mm)
A: Point Load (100 kN)	Solid Bar (Fe500)	92.79	0.527
A: Point Load (100 kN)	Optimized Hollow Bar (Fe500)	105.58	0.611
A: Point Load (100 kN)	Optimized Hollow Bar (CFRP)	92.35	0.700
A: Point Load (100 kN)	Optimized Hollow Bar (GFRP)	52.36	1.096
A: Point Load (100 kN)	Optimized Hollow Bar (Titanium)	80.67	0.801
B: UDL (10 kN/m)	Solid Bar (Fe500)	4.32	0.024
B: UDL (10 kN/m)	Optimized Hollow Bar (Fe500)	4.64	0.028
B: UDL (10 kN/m)	Optimized Hollow Bar (CFRP)	4.22	0.032
B: UDL (10 kN/m)	Optimized Hollow Bar (GFRP)	3.13	0.051
B: UDL (10 kN/m)	Optimized Hollow Bar (Titanium)	3.90	0.037
C: Varying Linear Load (0→20 kN/m)	Solid Bar (Fe500)	41.05	0.201
C: Varying Linear Load (0→20 kN/m)	Optimized Hollow Bar (Fe500)	45.01	0.227

Loading Condition	Reinforcement Type	Max Stress (MPa)	Max Deformation (mm)
C: Varying Linear Load (0→20 kN/m)	Optimized Hollow Bar (CFRP)	42.85	0.252
C: Varying Linear Load (0→20 kN/m)	Optimized Hollow Bar (GFRP)	33.21	0.371
C: Varying Linear Load (0→20 kN/m)	Optimized Hollow Bar (Titanium)	40.94	0.283
D: Combined Loading	Solid Bar (Fe500)	111.00	0.727
D: Combined Loading	Optimized Hollow Bar (Fe500)	126.84	0.838
D: Combined Loading	Optimized Hollow Bar (CFRP)	112.08	0.957
D: Combined Loading	Optimized Hollow Bar (GFRP)	78.45	1.486
D: Combined Loading	Optimized Hollow Bar (Titanium)	98.04	1.092

The identical set of four loading conditions summarised in Table 9 was applied to reinforced concrete beams incorporating solid and topology-optimised hollow pentagonal bars. Table 11 presents the corresponding results in placeholder form pending simulation.

Table 11: Maximum stress and deformation for reinforced concrete beams with pentagonal reinforcement

Loading Condition	Reinforcement Type	Max Stress (MPa)	Max Deformation (mm)
A: Point Load (100 kN)	Solid Bar (Fe500)	199.845	0.642
A: Point Load (100 kN)	Optimized Hollow Bar (Fe500)	127.86	0.807
A: Point Load (100 kN)	Optimized Hollow Bar (CFRP)	107.88	0.901
A: Point Load (100 kN)	Optimized Hollow Bar (GFRP)	57.02	1.231
A: Point Load (100 kN)	Optimized Hollow Bar (Titanium)	87.83	0.996
B: UDL (10 kN/m)	Solid Bar (Fe500)	3.79	0.029
B: UDL (10 kN/m)	Optimized Hollow Bar (Fe500)	5.12	0.037
B: UDL (10 kN/m)	Optimized Hollow Bar (CFRP)	4.79	0.042
B: UDL (10 kN/m)	Optimized Hollow Bar (GFRP)	3.39	0.058

Loading Condition	Reinforcement Type	Max Stress (MPa)	Max Deformation (mm)
B: UDL (10 kN/m)	Optimized Hollow Bar (Titanium)	4.49	0.047
C: Varying Linear Load (0→20 kN/m)	Solid Bar (Fe500)	40.71	0.240
C: Varying Linear Load (0→20 kN/m)	Optimized Hollow Bar (Fe500)	83.52	0.288
C: Varying Linear Load (0→20 kN/m)	Optimized Hollow Bar (CFRP)	78.44	0.315
C: Varying Linear Load (0→20 kN/m)	Optimized Hollow Bar (GFRP)	53.87	0.413
C: Varying Linear Load (0→20 kN/m)	Optimized Hollow Bar (Titanium)	71.07	0.343
D: Combined Loading	Solid Bar (Fe500)	121.28	0.883
D: Combined Loading	Optimized Hollow Bar (Fe500)	157	1.103
D: Combined Loading	Optimized Hollow Bar (CFRP)	136.72	1.227
D: Combined Loading	Optimized Hollow Bar (GFRP)	96.50	1.666
D: Combined Loading	Optimized Hollow Bar (Titanium)	127.32	1.353

3.6 Environmental Impact Assessment

The topology-optimised hollow hexagonal and pentagonal reinforcement bars achieved approximately 20% material reduction, leading to lower raw material consumption, embodied energy, and associated carbon emissions. Reduced material usage also decreases transportation requirements and construction waste while maintaining satisfactory structural performance. These findings demonstrate that topology optimisation enhances the sustainability of reinforced concrete structures by improving resource efficiency and reducing the overall environmental footprint without compromising structural integrity.

3.7 Cost Comparison

Because the side length of both the hexagonal and pentagonal reinforcement bars was fixed at 20 mm, their cross-sectional areas differ, resulting in different material consumption per unit length for the two geometries. Consequently, the unit material cost per metre depends on the respective cross-sectional area and the material selected. The percentage cost saving achievable through the hollow, topology-optimised configuration is directly proportional to the percentage material reduction determined using Equation (17). The specific unit costs, cost savings, and projected annual savings for the hexagonal and pentagonal reinforcement bars are based on current market prices and the corresponding topology-optimised designs and are presented in Table 12.

Table 12: Results of the cost analysis for the hexagonal and pentagonal reinforcement bars

Geometry	Material	Cost of Solid Rebar (Sq/m)	Cost of Optimized Hollow Rebar (Sq/m)	Cost Saving (%)	Estimated Annual Savings (per 1000 m)
Hexagonal	Fe500 Steel	143.45	114.76	20	28,690
	CFRP	1,132.50	906	20	2,26,500
	GFRP	528.5	422.8	20	1,05,700
	Titanium	2,718.00	2,174.40	20	5,43,600
Pentagonal	Fe500 Steel	95	76	20	19,000
	CFRP	750	600	20	1,50,000
	GFRP	350	280	20	70,000
	Titanium	1,800.00	1,440.00	20	3,60,000

The figure 9 shows the cost comparison of solid and optimized hollow hexagonal reinforcement bars from various materials. With the optimized hollow design, the cost of the product is reduced consistently by 20 % compared to the same product made in solid bar. Fe500 steel decreases from ₹143.45/m to ₹114.76/m, CFRP from ₹1132.50/m to ₹906.00/m, GFRP from ₹528.50/m to ₹422.80/m, and Titanium from ₹2718.00/m to ₹2174.40/m. While titanium is still the most costly material, it also gives the greatest absolute cost saving, showing that topology optimization can also reduce material usage and overall manufacturing costs without affecting structural efficiency.

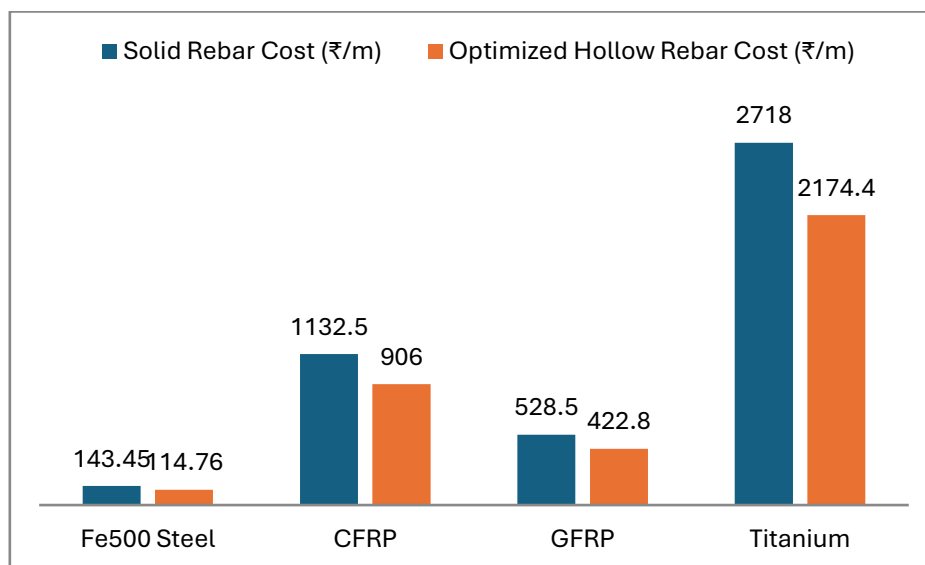


Figure 9: Cost comparison of solid and hollow hexagonal rebars

The figure 10 is the approximate annual cost savings for the optimized hollow reinforcement bar as against the length of reinforcement of 1000 m. Results show that the hexagonal geometry always yields greater savings than the pentagonal

geometry for all materials. For a Hexagonal geometry, Fe500 steel has saved of ₹28,690 and CFRP has saved of ₹2,26,500 whereas, in a Pentagonal geometry, Fe500 steel has saved of ₹19,000 and CFRP has saved of ₹1,50,000. GFRP provides savings of ₹1,05,700 and ₹70,000, whereas Titanium delivers the highest savings of ₹5,43,600 and ₹3,60,000.

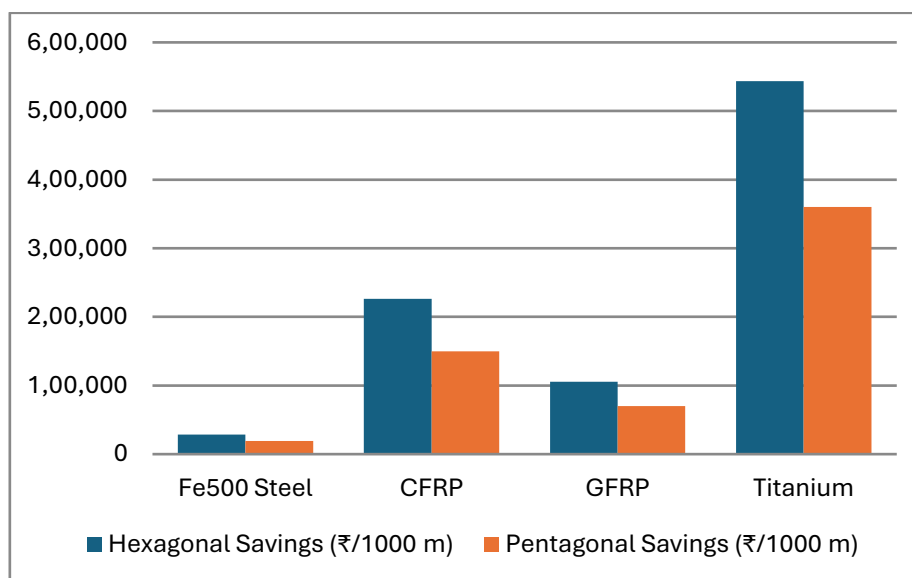


Figure 10: Annual cost savings of optimized hexagonal and pentagonal rebars.

CONCLUSIONS

The findings in this work showed that topology optimization provide better economic benefits in all reinforcement materials in the development of lightweight hexagonal and pentagonal reinforcement bars than FEA. The optimized hollow configurations resulted in about 20% reduction in material and cost and still had acceptable structural performance under 10 kN of axial tensile load and various beam loading tests. In the case of hexagonal reinforcement, the stress increased slightly, ranging from 12.76% to 18.97%, where the Fe500 steel had a stress of 24.8 MPa and a deformation of 0.031 mm, which is excellent stiffness. The stress increase of the pentagonal hollow bars was, on the other hand, larger at 70.54–82.72%, with the Fe500 steel being 44.4 MPa and 0.0601 mm. The optimized Fe500 hexagonal reinforcement under combined beam loading, had the maximum stress of 126.84MPa, and deformation of 0.838mm, all within acceptable structural limits. From the economical point of view optimized design shows cost saving of up to ₹ 5,43,600 per 1000m for Titanium and ₹ 2,26,500 for CFRP, Fe500 steel showing optimized view with a balance of cost, strength, and durability. In general, the proposed polygonal reinforcement bars (PRB) provide a structurally safe, sustainable, and economical solution for reinforced concrete structures, compared with traditional monolithic reinforcement.

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