Natural Resources for Human Health
eISSN: 2583-1194

Original Research

Received 26 May 2026
Revised 30 June 2026
Accepted 15 July 2026

Natural Resources for Human Health 2026, Vol. 6 (10s): 19–34

KEYWORDS:
Bank sector; Geometric Brownian motion; Stochastic Volatility; Forecasting index prices

Using GBM and GFBM to Forecast the Performance of the Bank Sector in Saudi Arabia.

Mustafa Ahmed Hamad Mansour

Prince Sattam bin Abdulaziz University, Saudi Arabia
EMAIL: rcb.mansour@gmail.com
m.mansour@psau.edu.sa
https://orcid.org/0009-0004-2251-6152

ABSTRACT: The future index prices are considered a central issue because investors and traders hope to know the future performance of the market. Numerous models have developed for this purpose. Geometric Brownian motion (GBM) is well-known model. This study investigates four types of GBM based on the existence of memory or the type of volatility. These models involve classical GBM, SV GBM, GFBM, and SV GFBM. These models were employed in an empirical study to forecast the future index prices of three banks in the Saudi Stock Exchange Market; Rajhi Bank, Riyadh Bank, and Ahli Bank. The evaluation was conducted using two criteria: mean square error (MSE) and mean absolute percentage error (MAPE). The results show that the smallest MSE and MAPE values were obtained for the SV GFBM for the three banks. These outcomes have confirmed the direct positive effect of taking memory and stochastic volatility assumptions into account in the GBM model, which was confirmed by some works. In general, all models considered in this work exhibited high accuracy with MAPE value of $\leq$ 10%. This finding supports the ability of these models to apply in a real financial environment. Through this study, we can conclude that the banking sector in Saudi Arabia is stable and predictable, and we encourage investors to trade in this sector.

INTRODUCTION

The banking sector is important in the Saudi Exchange stock market. It consists of eleven banks with a total capital of 225.8 billion riyals ($60.2 billion), which represents 31% of the companies listed on the Saudi Exchange stock market. This means that, there are a large slice of investors and speculators trading in this sector. Therefore, forecasting the future performance of the banking sector is crucial issue for individual investors, trading companies, and banks. Where every trader aims to avoid losing and gaining profits. In addition, forecasting can help investors decide how to allocate their business budgets and plan for expected expenses in the upcoming period. For this purpose, many models have been proposed to forecast future prices based on historical data such as random walk, jump diffusion, Brownian motion processes, geometric Brownian motion (GBM), and Geometric Fractional Brownian Motion (GFBM).

This work investigates the GBM and GFBM models and employs them to forecast future share prices for three banks in Saudi Arabia: Al Rajhi Bank, Riyadh Bank, and Ahli Bank. The chosen GBM and GFBM models depend on certain properties of the Brownian motion process. Although Brownian motion is continuous everywhere (like the movement of prices), it is not differentiable anywhere. The BM is self-similar; that is, if any small part of the Brownian motion time series trajectory is expanded, then it looks like the entire trajectory. Once Brownian motion reaches zero or any specific value, it l returns to that value infinitely many times in the future. The BM process is defined as follow.

Definition 1: *A Wiener Process or Brownian motion (BM) is a stochastic process* B_t *that satisfies the next conditions:*

i. B_t $is\ a\ continuous\ function\ of\ time\ with\ B_0 = 0.$

ii. B_t *has independent increments* ($B_t - B_s$ *and* $B_v - B_u$ *are independent for all* $v > v > t > s$).
iii. $B_t - B_s \sim N(0, t-s)$ *for all* $t > s$.

The BM will eventually reach any real value, regardless of whether it is large or negative. It may be very high above the axis at some point, but will return to zero at some point in the future. Ross (1999) utilized the BM with its properties to model stock prices directly. However, Ross's work faced logical criticism because it permits the price to be negative because the stock price is supposed to be a normal random variable. To enhance the financial application of BM and overcome the problem of price negativity, Geometric Brownian motion (GBM) was presented. GBM can be defined as a non-negative variation in the BM. Therefore, GBM can be applied in many financial applications such as the famous Black- Sholes model of option pricing index prices, value at risk, exchange rate, and mortgage insurance. The GBM is useful in modeling random price fluctuations over time and in studying the performance of commodity prices. Hence, the GBM is considered a useful model for price prediction. The GBM is defined as follows:

Definition 2: *The stochastic process* X_t *is said to follow GBM if it satisfies the next stochastic differential equation (SDE):*

$$dX_t = \mu X_t dt + \sigma X_t dB_t, \tag{1}$$

where B_t *is a* BM, μ and σ *are drift and volatility respectively. The general solution of this SDE is given by*

$$X_t = X_0\, exp\left\{\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma B_t\right\} \tag{2}$$

where X_0 *is an initial value.*

GBM assumes that the time series of price changes is independent of previous prices. Meanwhile, GBM's wide-reaching applications in financial situations have attracted the attention of many researchers for the presence of memory in GBM-controlled time series data (see, for example, Painter (1998), Grau-Carles (2000), Rejichi (2012), Alhagyan (2018), Al-Hajjaryan & Yassen (2022), Abbas (2023), Kim (2022), Han (2022), etc.). Hence, the need to incorporate memory properties into GBM has led to the development of a model for GBM called Geometric Fractional Brownian Motion (GFBM), which is obtained by replacing the BM process in the GBM with the FBM process. The FBM is a continuous Gaussian process with independent increments defined as follows.

Definition 3: *The fractional Brownian motion* $(FBM), \{B_H(t)\}$,*with Hurst parameter* $H \in (0,1)$ *is a centered Gaussian process whose paths are continuous with probability 1 and its distribution is defined by the covariance structure:*

$$E[B_H(t)B_H(s)] = \tfrac{1}{2}(t^{2H} + s^{2H} - |t-s|^{2H}).$$

The correlation between the increments of FBM fluctuated consistently with the Hurst parameter (or self–similarity) parameter H. Hurst parameter name comes from the name of researcher Harold Edwin Hurst (1880–1978) who examined the erratic rainfall and drought circumstances along the Nile River over a long period. According to the value of H , there are three types of memory dependence: short-memory dependence $(0 < H < 0.5)$, no memory dependence $(H = 0.5)$, and long-memory dependence $(0.5 < H < 1)$. The GFBM model is defined as follows:

Definition 4: *The stochastic process* X_t *is said to follow GFBM if it satisfies the next (SDE):*

$$dX_t = \mu X_t dt + \sigma X_t\, dB_{H_1}(t) \tag{3}$$

where $B_{H_1}(t)$ *is FBM,μ and σ are drift and volatility respectively. The solution is given by*

$$X(t) = X_0\, exp\left[\left(\mu\ - \tfrac{1}{2}\sigma^2 t^{2H_1 - 1}\right)t + \sigma B_{H_1}(t)\right], \tag{3}$$

where X_0 *is an arbitrary initial value.*

Several applications of the GFBM have been reported in the literature. To just a few, option pricing (Alhagyan 2018, Alhagyan et al. 2016 a; 2016 b, Misiran et al. 2010; 2012 and Kukush et al. 2005), index prices (Abbas & Alhagyan 2022; 2023, Alhagyan 2018, Xiao et al. 2015 and Alhagyan & Al-Duais 2020), value at risk (Alhagyan et al. 2021, Alhagyan 2018, and Wang et al.

2017), exchange rate (Alhagyan 2018, Alhagyan 2022, and Gözgör 2013), and mortgage insurance (Bardhan et al. 2006, Alhagyan 2018, Chen et al. 2013, and Alhagyan et al. 2021).

Many researchers have used the assumption of constant volatility (σ) in the GBM and GFBM to simplify complex computations. However, many researchers have not supported this assumption and encouraged modeling GBM and GFBM under the assumption of stochastic volatility (SV) by the premise that stochastic volatility can explain the real financial situation more precisely than constant volatility (Stein 1989, Bakshi et al. 2000, and Aït–Sahalia and Lo 1998). In the stochastic volatility (SV) model the constant volatility σ in GBM and GFBM is replaced by a deterministic function of a stochastic process or volatility process $\sigma(Y_t)$ where Y_t is the solution of other stochastic differential equations (SDE) that are driven by other noise stochastic volatilities. Table 1 shows different SDE models that describe Y_t in different forms.

The ability of the SV model to capture the effect of time–varying volatility makes it suitable for financial markets. In the SV model, the volatility and common dependence between variables are permitted to fluctuate over time instead of remaining constant. This indicates that there are two sources of randomness in the SV model, that may or may not be correlated. The definitions of the SV models under study are as follows: GBM perturbed by FOU and GFBM perturbed by FOU.

Table 1. Models of stochastic processes describing Y_t in SV models

Name	Model
$\boldsymbol{Log-normal\ process}$	$dY_t = \alpha Y_t dt + \beta Y_t dB_{2t}$
$\boldsymbol{Cox-Ingersoll-Ross\ (CIR)\ process}$	$dY_t = \theta(\omega - Y_t)dt + \xi\sqrt{Y_t}dB_{2t}$
$\boldsymbol{Ornstein-Uhlenbeck\ (OU)\ process}$	$dY_t = \alpha(m - Y_t)dt + \beta dB_{2t}$
$\boldsymbol{Not\ mean\ reverting\ process}$	$dY_t = \alpha Y_t dB_{2t}$
$\boldsymbol{Fractional\ Ornstein-Uhlenbeck\ (FOU)\ process}$	$dY_t = \alpha(m - Y_t)dt + \beta dB_{H_2}(t)$

The ability of SV model to capture the effect of time–varying volatility makes it considerable for financial markets. In SV model the volatility and common dependence between variables are permitted to fluctuate over time, instead of remaining constant. This indicates that there are two sources of randomness in the SV model, which may or may not be correlated. In the following, the definitions of SV models under study are presented; GBM perturbed by FOU and GFBM perturbed by FOU.

Definition 5: *The stochastic process X_t is said to follow GBM perturbed by SV (FOU) if it satisfies the next SDE:*

$$dX_t = \mu X_t dt + \sigma(Y_t) X_t dB_{1t} \tag{5}$$

where μ is a mean of return, Y_t is a stochastic process, B_{1t} is a BM, and $\sigma(Y_t) = Y_t$ is a deterministic function. Let the dynamics of volatility Y_t follow fractional Ornstein–Uhlenbeck (FOU) process which is the solution of the following SDE:

$$dY_t = \alpha(m - Y_t)dt + \beta dB_{H_2}(t), \tag{6}$$

where a, β and m are mean reverting of volatility, volatility of volatility, and mean of volatility, respectively. $B_{H_2}(t)$ is a FBM which is independent from B_{1t}.

Definition 6: *A stochastic process X_t is said to follow a GFBM perturbed by SV (FOU) if it satisfies the following SDE:*

$$dX_t = \mu\, X_t dt + \sigma(Y_t) X_t dB_{H_1}(t), \tag{7}$$

where μ is mean of return, Y_t is a stochastic process, $B_{H_1}(t)$ is a FBM with Hurst index

H_1, and $\sigma(Y_t) = Y_t$ is a deterministic function. Let the dynamics of volatility Y_t be described by fractional Ornstein-Uhlenbeck (FOU)process which is solution of the following SDE

$$dY_t = \alpha(m - Y_t)dt + \beta dB\_(H_2)\,(t), \tag{8}$$

where α, β and m are constant parameters that represent mean reverting of volatility,

volatility of volatility, and mean of volatility, respectively. $B_{H_2}\ (t)$ *is a FBM which is independent from* $B_{H_1}\ (t)$.

Forecasting

This work employs four models (as mentioned before in Definitions 2, 4, 5, and 6) to forecastthe future index prices of three Banks in Saudi Arabia, including Al Rajhi Bank, Riyadh Bank, and Ahli Bank. This study examines the effect of incorporating the properties of memory and stochastic volatility into the GBM model. Historical price data on stock performance are used to forecast future stock prices for every bank separately.

To evaluate and compare the performance of each model, two measures of error were used: the mean square error (MSE) and the mean absolute percentage error (MAPE).

$$\text{MSE}=\frac{\sum_{i=1}^{n}(Y_i-F_i)^2}{n}\ \&\ \text{MAPE}=\frac{\sum_{i=1}^{n}\frac{|Y_i-F_i|}{Y_i}}{n},$$

where F_i and Y_i represent the forecast and actual prices on day i, respectively. While n the total number of forecast days. Lawrence (2009) provides the following table to judge the forecasting method using the MAPE.

Table 2. MAPE to judgment accuracy of forecasting method.

Accuracy	MAPE
Highly accurate	MAPE $< 10\%$
Good accurate	$10\% \leq$ MAPE $< 20\%$
Reasonable	$20\% \leq$ MAPE $< 50\%$
Inaccurate	MAPE $\geq 50\%$

Al Rajhi Bank

Description of Historical Data

Historical data are available online at *https://www.saudiexchange.sa*. The total number of working days is 23, beginning from 1st August 1, 2023, to 30th August 30, 2023. The return series is considered in logarithm form (i.e., $r_n = \ln(S/S_{n-1})$) to deal with high volatility in the data. Figure 1 and 2 show the closing prices and their return series.

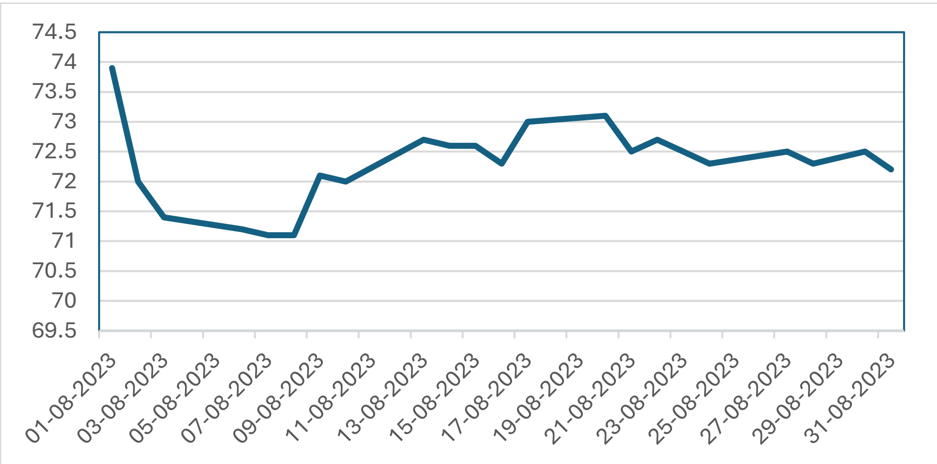


Figure 1. Daily close price of Rajhi Bank.

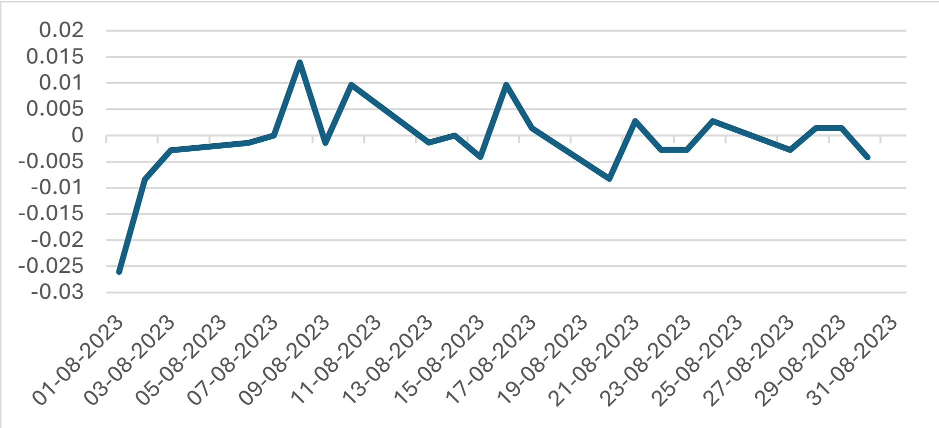


Figure 2. Daily returns of Rajhi Bank.

Forecasting and Evaluation

According to the historical data of Rajhi Bank, all parameters involved in all models under study are computed using Mathematica 10 software and then employed to calculate both constant volatility and stochastic volatility. Table 3 lists all the computed parameters and volatilities.

Table 3. Parameters and volatilities values of Rajhi Bank.

Parameter	Value	Parameter	Value
H_1	0.59061	β	0.00014
H_2	0.25151	m	0.00006
μ	-0.00106	α	1.62712
Computed Volatility			
σ	0.00776		
$\sigma(Y_t)$	0.00575		

These parameters are exploited to forecast the close price of the next month 2023 (September). The forecasting is computed using four models; GBM, GFBM, GBM perturbed by SV (SV GBM) and GFBM perturbed by SV (SV GFBM) as mentioned in Definitions 2, 4, 5 and 6. Table 4 shows the forecasted prices in addition to the actual prices of Rajhi Bank. Meanwhile, Tables 5 and 6 show the level of accuracy of each model.

Table 4. Actual and forecasted prices of Rajhi Bank.

Date	Actual	GBM	GFBM	SV GBM	SV GFBM
9/3/2023	72.200	72.146	72.219	72.291	72.203
9/4/2023	71.900	72.175	72.167	72.319	72.194
9/5/2023	72.100	72.191	72.185	72.335	72.196
9/6/2023	71.900	72.168	72.175	72.313	72.195
9/7/2023	70.800	72.203	72.229	72.347	72.174
9/10/2023	70.200	72.145	72.189	72.289	72.196
9/11/2023	70.800	72.172	72.183	72.316	72.175
9/12/2023	70.900	72.169	72.157	72.313	72.190
9/13/2023	70.800	72.194	72.202	72.339	72.178

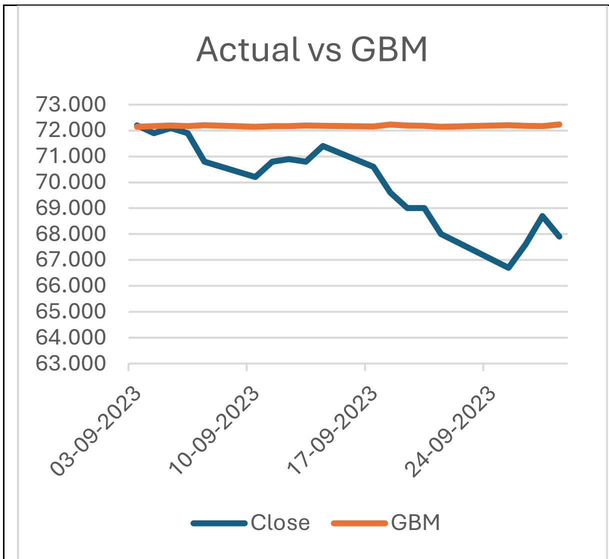
9/14/2023	71.400	72.183	72.174	72.327	72.193
9/17/2023	70.600	72.161	72.201	72.306	72.197
9/18/2023	69.600	72.228	72.186	72.373	72.174
9/19/2023	69.000	72.198	72.174	72.343	72.192
9/20/2023	69.000	72.182	72.211	72.326	72.178
9/21/2023	68.000	72.148	72.177	72.292	72.192
9/25/2023	66.700	72.209	72.161	72.354	72.189
9/26/2023	67.600	72.179	72.226	72.324	72.180
9/27/2023	68.700	72.167	72.172	72.312	72.190
9/28/2023	67.900	72.236	72.205	72.380	72.176

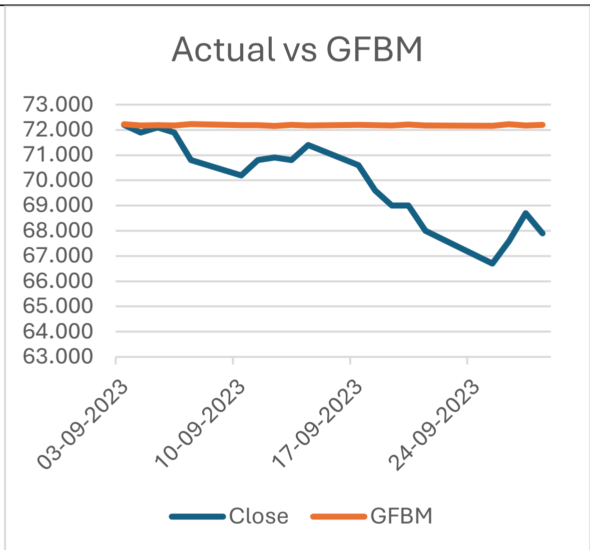
Table 5. The accuracy ranking level of forecasting model based on MSE (Rajhi Bank).

Rank	Model	MSE
1	SV GFBM	7.4078
2	GFBM	7.4211
3	GBM	7.4268
4	SV GBM	8.0707

Table 6. The accuracy ranking level of forecasting model based on MAPE (Rajhi Bank).

Rank	Model	MAPE
1	SV GFBM	3.16 %
2	GFBM	3.17 %
3	GBM	3.18 %
4	SV GBM	3.37 %





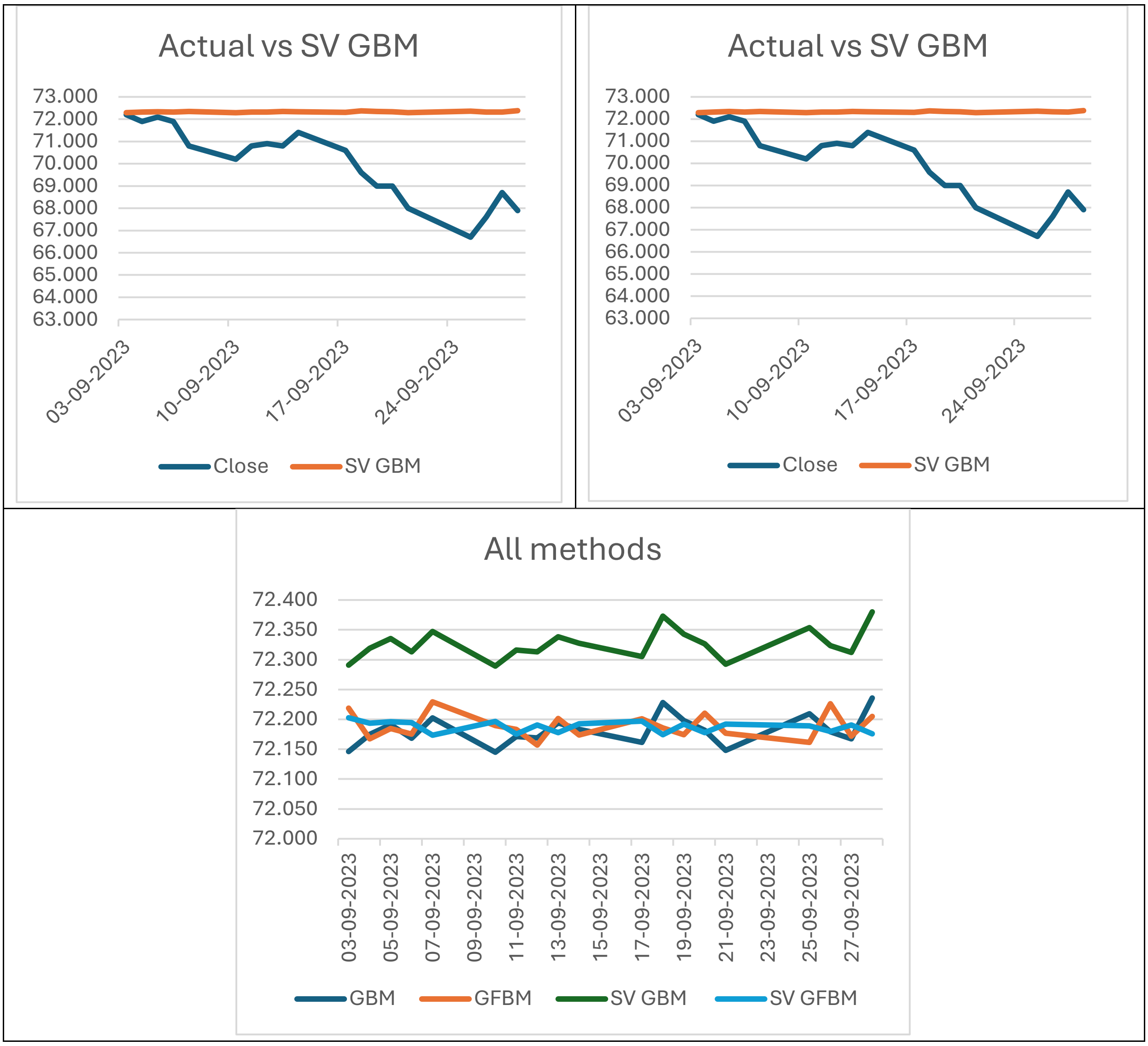


Figure 3. Actual prices vs Forecast prices (Rajhi Bank).

Riyadh Bank

Description of Historical Data

Historical data are available online at *https://www.saudiexchange.sa*. The total number of working days is 23, beginning from 1st August 1, 2023, to 30th August 30, 2023. The return series is considered in logarithm form (i.e., $r_n = \ln(S/S_{n-1})$) to deal with high volatility in the data. Figure 4 and 5 show the closing prices and their return series.

According to the historical data of the Riyadh Bank, all parameters involved in all the models under study were computed using Mathematica 10 software and then employed to calculate both constant volatility and stochastic volatility. Table 7 lists all the computed parameters and volatilities.

Table 7. Parameters and volatilities values of Riyadh Bank.

Parameter	Value	Parameter	Value
H_1	0.26908	β	0.00016
H_2	0.47924	m	0.00015

μ	-0.00147	α	1.49762
Computed Volatility			
σ	0.01256		
$\sigma(Y_t)$	0.01490		

These parameters are exploited to forecast the close price of the next month 2023 (September). The forecasting is computed using four models; GBM, GFBM, GBM perturbed by SV (SV GBM) and GFBM perturbed by SV (SV GFBM) as mentioned in Definitions 2, 4, 5 and 6. Table 8 shows the forecasted prices in addition to the actual prices of Riyadh Bank. Meanwhile, Tables 9 and 10 show the level of accuracy of each model.

That is what the findings showed, in light of the smallest MSE and MAPE values, the SV GFBM accomplished ahead of all models in terms of exactness. This outcome was a result of the presence of two provenances of memory with stochastic volatility incorporated into the SV GFBM model. SV GBM was put last despite the presence of two provenances of randomness but with the assumption of constant volatility. Table 2 shows that all the models had high accuracy, with MAPE <10%. In addition, the MSE values of SV GFBM, GBM, and GFBM were close to one another, whereas the SV GBM was moderately far. The outcomes supported that models with memory (SV GFBM and GFBM) were more appropriate for forecasting future stock prices than models without memory (SV GBM and GBM). This result confirms the findings of many empirical studies, such as Abbas and Alhagyan (2022 and 2023), Mansour and Alhagyan (2022), Alhagyan and Yassen (2023), Alhagyan (2022), Willinger et al. (1999), Painter (1998), Rejichi & Aloui (2012), and Alhagyan & Alduais (2020). Figure 6 illustrates the comparison between the actual and forecasted closing prices.

Table 8. Actual and forecasted prices of Riyadh Bank.

Date	Actual	GBM	GFBM	SV GBM	SV GFBM
9/3/2023	30.05	30.0626	30.0386	30.1231	30.0463
9/4/2023	30.00	30.0584	30.047	30.1186	30.0485
9/5/2023	29.95	30.0397	30.0953	30.0999	30.0616
9/6/2023	29.35	30.0601	30.0593	30.1203	30.0516
9/7/2023	29.00	30.0496	30.0531	30.1093	30.0498
9/10/2023	28.70	30.0552	30.0624	30.1151	30.0522
9/11/2023	29.10	30.0917	30.0454	30.152	30.0474
9/12/2023	29.30	30.0494	30.0276	30.11	30.0423
9/13/2023	29.30	30.0645	30.0541	30.1243	30.0495
9/14/2023	28.90	30.0381	30.0306	30.0984	30.043
9/17/2023	28.55	30.0381	30.0306	30.0984	30.043
9/18/2023	28.55	30.0722	30.0769	30.1326	30.0555
9/19/2023	28.60	30.0819	30.0654	30.1419	30.0522
9/20/2023	28.05	30.0474	30.0438	30.1075	30.0462
9/21/2023	28.00	30.0564	30.0555	30.1166	30.0492
9/25/2023	28.00	30.0596	30.0326	30.1194	30.0429
9/26/2023	27.40	30.0279	30.0413	30.088	30.0452

Natural Resources for Human Health
eISSN: 2583-1194

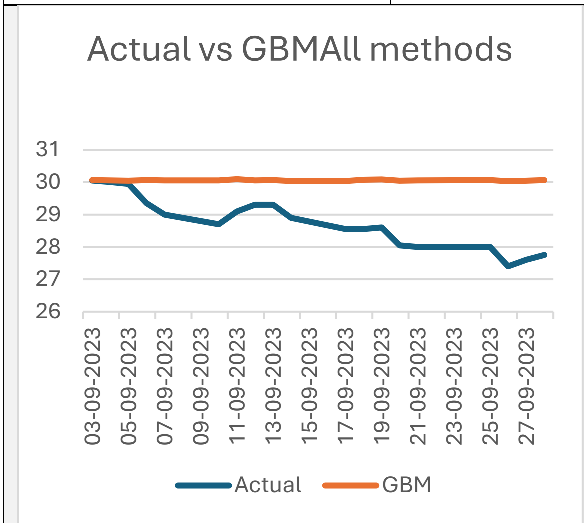
9/27/2023	27.60	30.0473	30.0394	30.1074	30.0445
9/28/2023	27.75	30.0598	30.0499	30.1203	30.0472

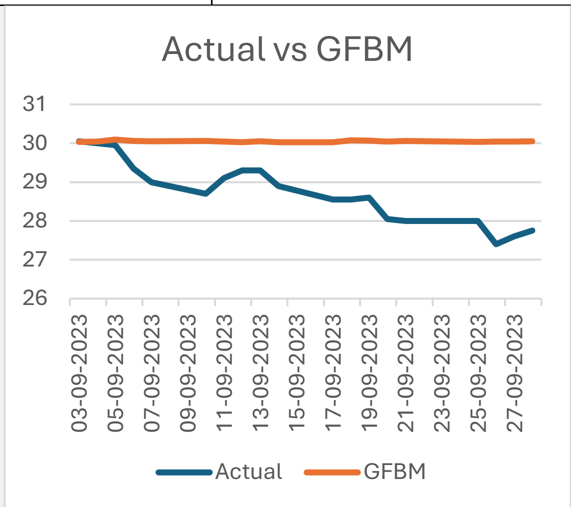
Table 9. The accuracy ranking level of forecasting model based on MSE (Riyadh Bank).

Rank	Model	MSE
1	SV GFBM	2.3140
2	GFBM	2.3148
3	GBM	2.3314
4	SV GBM	2.4928

Table 10. The accuracy ranking level of forecasting model based on MAPE (Riyadh Bank).

Rank	Model	MAPE
1	SV GFBM	4.61 %
2	GFBM	4.62 %
3	GBM	4.64 %
4	SV GBM	4.85 %





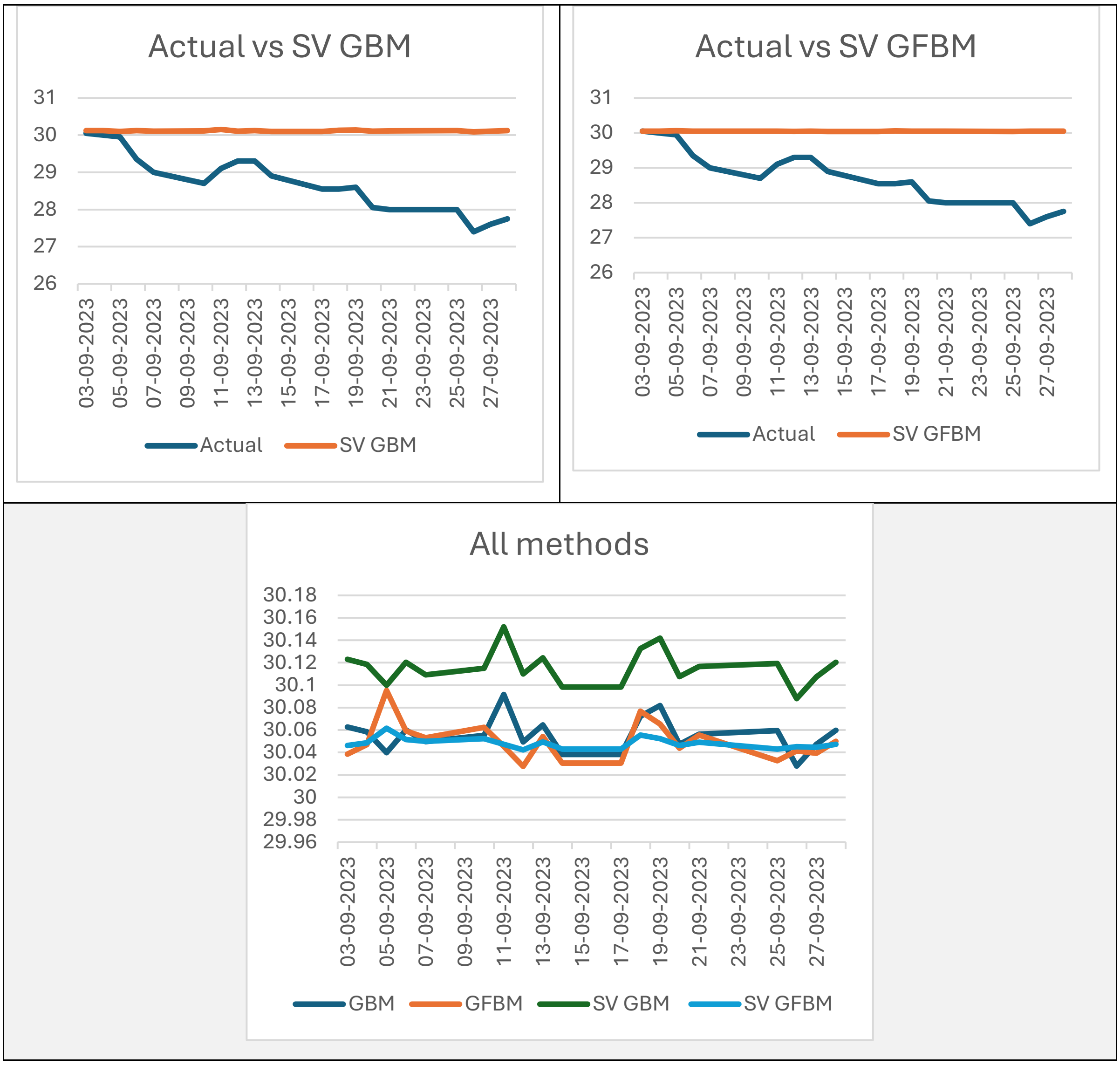


Figure 6. Actual prices vs Forecast prices (Riyadh Bank).

Ahli Bank

Description of historical data

Historical data are available online at *https://www.saudiexchange.sa*. The total number of working days is 23, beginning from 1st August 1, 2023, to 30th August 30, 2023. The return series is considered in logarithm form (i.e., $r_n = \ln(S/S_{n-1})$) to deal with high volatility in the data. Figure 7 and 8 show the closing prices and their return series.

Natural Resources for Human Health
eISSN: 2583-1194

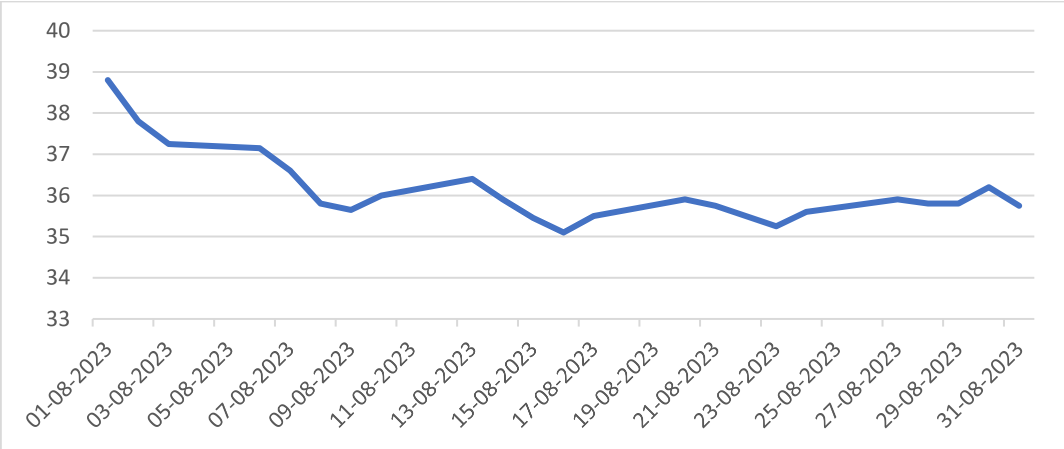


Figure 7. Daily close price of Ahli Bank.

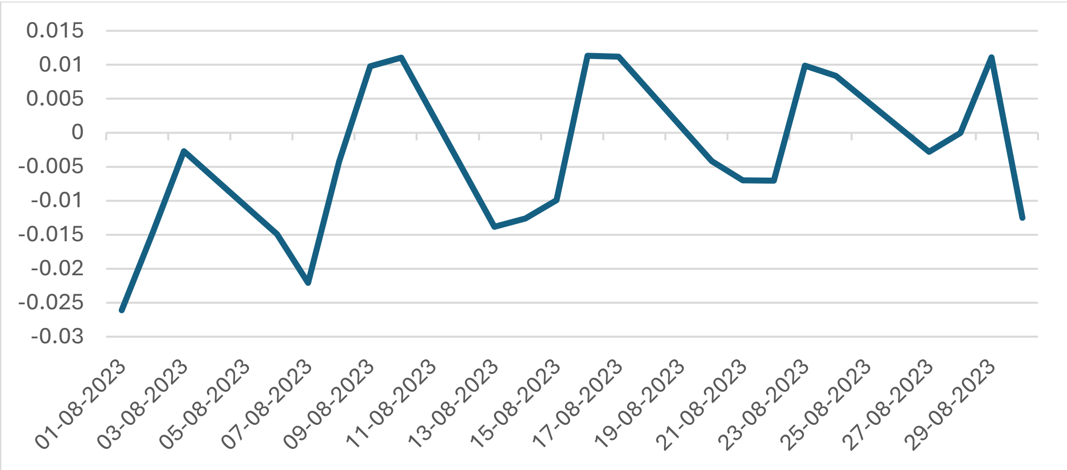


Figure 8. Daily returns of Ahli Bank.

Forecasting and Evaluation

According to the historical data of Ahli Bank, all parameters involved in all models under study are computed using Mathematica 10 software and then employed to calculate both constant volatility and stochastic volatility. Table 11 lists all computed parameters and volatilities.

Table 11. Parameters and volatilities values of Ahli Bank.

Parameter	Value	Parameter	Value
$\boldsymbol{H_1}$	0.70769	β	0.00013
$\boldsymbol{H_2}$	0.75185	m	0.00013
$\boldsymbol{\mu}$	-0.00372	α	1.62712
Computed Volatility			
$\boldsymbol{\sigma}$	0.01159		
$\boldsymbol{\sigma(Y_t)}$	0.01279		

These parameters are exploited to forecast the close price of the next month 2023 (September). The forecasting is computed using four models; GBM, GFBM, GBM perturbed by SV (SV GBM) and GFBM perturbed by SV (SV GFBM) as mentioned in Definitions 2, 4, 5 and 6. Table 12 shows the forecasted prices in addition to the actual prices of Ahli Bank. Meanwhile, Tables 13 and 14 show the level of accuracy of each model.

Table 12. Actual and forecasted prices of Ahli Bank.

Date	Actual	GBM	GFBM	SV GBM	SV GFBM
9/3/2023	35.90	35.7391	35.7615	35.8103	35.752
9/4/2023	35.95	35.7422	35.7405	35.8138	35.7463
9/5/2023	35.60	35.7252	35.7339	35.7967	35.7443
9/6/2023	35.25	35.7795	35.7658	35.8506	35.752
9/7/2023	35.10	35.7546	35.7663	35.8259	35.7517
9/10/2023	34.90	35.7292	35.7649	35.8002	35.7509
9/11/2023	34.85	35.7687	35.7434	35.8408	35.7451
9/12/2023	34.55	35.7418	35.7687	35.8133	35.7512
9/13/2023	34.25	35.7575	35.777	35.8289	35.7528
9/14/2023	34.05	35.7344	35.7574	35.8058	35.7475
9/17/2023	34.00	35.7578	35.7199	35.8289	35.7376
9/18/2023	33.60	35.7752	35.7399	35.8463	35.7423
9/19/2023	33.60	35.7481	35.7682	35.8195	35.7491
9/20/2023	33.45	35.7228	35.7462	35.7946	35.7431
9/21/2023	33.10	35.7379	35.7322	35.8094	35.7391
9/25/2023	32.80	35.7175	35.7305	35.7887	35.7384
9/26/2023	32.80	35.7703	35.7495	35.8422	35.7427
9/27/2023	32.90	35.7404	35.7511	35.8119	35.7428
9/28/2023	32.80	35.7332	35.7887	35.8047	35.7519

Table 13. The accuracy ranking level of forecasting model based on MSE (Ahli Bank).

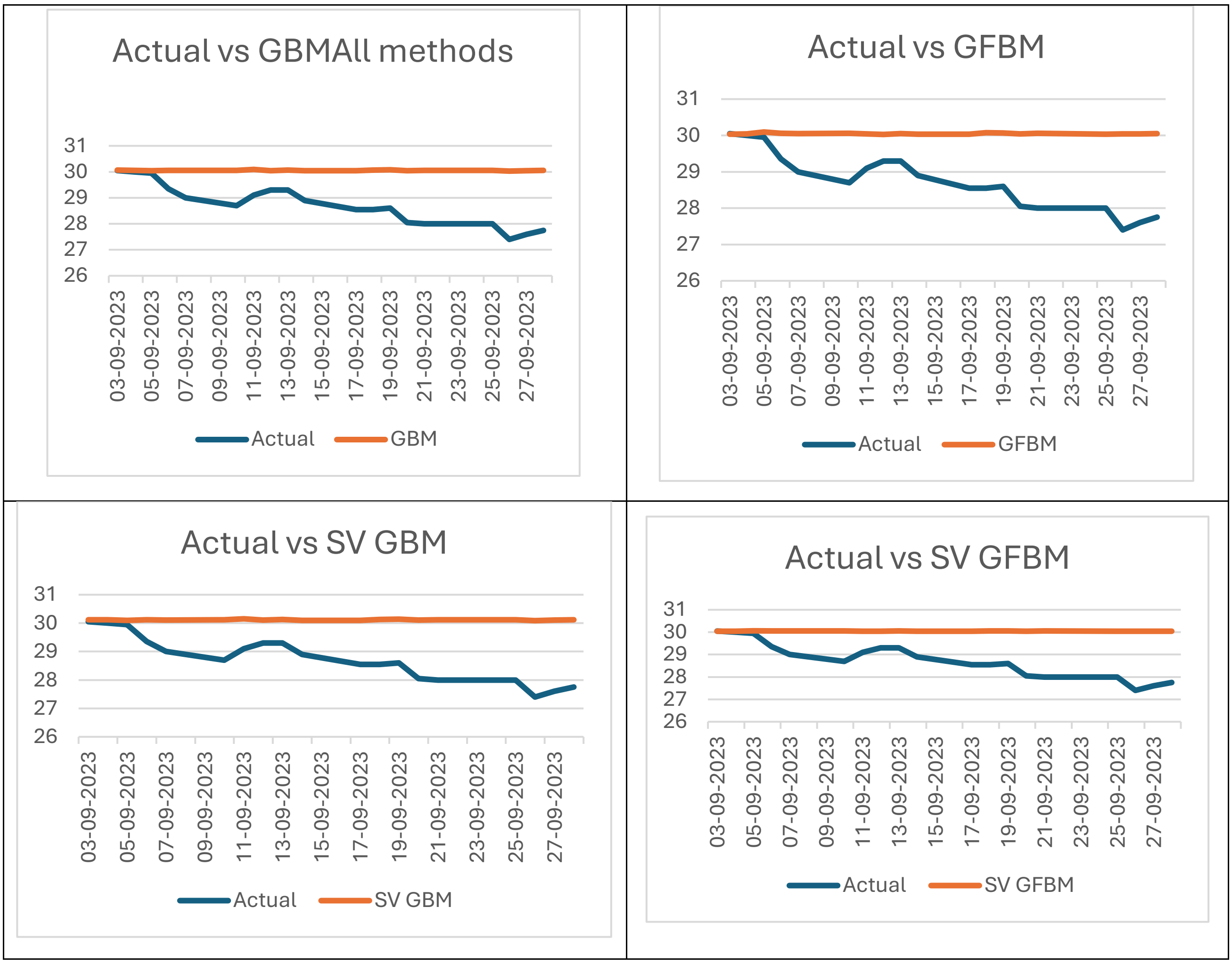
Rank	Model	MSE
1	SV GFBM	3.54781
2	GFBM	3.54848
3	GBM	3.57143
4	SV GBM	3.77717

Table 14. The accuracy ranking level of forecasting model based on MAPE (Ahli Bank).

Rank	Model	MAPE
1	SV GFBM	4.779%
2	GFBM	4.783%
3	GBM	4.797%
4	SV GBM	4.951%

Natural Resources for Human Health
eISSN: 2583-1194

Again, the findings exhibited that SV GFBM attained the first level in accuracy based on the smallest values of MSE and MAPE. This result was attained under the existence of two memory sources and stochastic volatility assumption in the SV GFBM model. Despite the existence of two sources of randomness, SV GBM attained the last level in accuracy but assumption of constant volatility. All models achieved MAPE <10% which means high accuracy in predicting according to Table 2. Additionally, the MSE values of SV GFBM, GBM, and GFBM were near to each other, but SV GBM was comparatively far away. The outcomes confirmed that the memory-rich models (SV GFBM and GFBM) were superior to the memory-poor models (SV GBM and GBM) in terms of their ability to predict future stock prices. This result is supported by a number of empirical findings, including those from Painter (1998), Alhagyan & Alduais (2020), Alhagyan & Yassen (2023), Abbas & Alhagyan (2022; 2023), and Alhagyan (2022), Painter (1998), and Rejichi & Aloui (2012). Figure 9 illustrates the comparison between the actual close prices versus forecasted close prices.



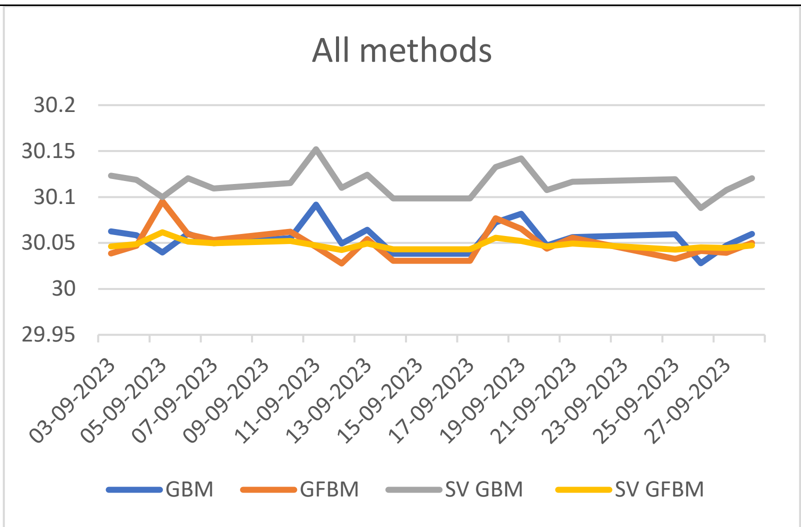


Figure 9. Actual prices vs Forecast prices (Ahli Bank).

CONCLUSION

A company's stock index price is a good indicator of its financial stability and rate of economic growth. Therefore, understanding and projecting the future direction of index prices is one of the top responsibilities of both stakeholders and investors. Many models have been proposed for this purpose. GBM is a significant model. Two assumptions—the constant assumption and stochastic volatility assumption—are used in the literature to discuss the volatility parameters that involve GBM. In addition, there are two hypotheses on the presence of memory in time-series financial data: one assumes that memory does not exist, while the other assumes that memory does. Consequently, this study investigated four models to cover the assumptions of memory and stochastic volatility. These models are the GBM (with assumptions of memoryless and constant volatility), SV GBM (with assumptions of memoryless and stochastic volatility), GFBM (with assumptions of memory and constant volatility), and SV GFBM (with assumptions of memory and stochastic volatility). The investigation was implemented using four models to forecast future index prices for three banks (Alrajhi Bank, Riyadh Bank, and Ahli Bank) in the Saudi Stock Exchange Market. The evaluation was conducted using two statistical criteria, MSE and MAPE. The results of this empirical investigation revealed that the SV GFBM achieved the best performance, depending on the smallest values of both MSE and MAPE for all three banks. SV GBM ranked last despite the existence of memory but with a constant volatility assumption. The outcomes support that GFBM models are more appropriate than GBM models for forecasting future stock prices under the same assumption of volatility. This outcome proves that incorporating memory and stochastic volatility together into GBM has a positive effect.

Moreover, the values of the MSE and MAPE for SV GFBM, GBM, and GFBM were relatively close, whereas the SV GBM was moderately far. Generally, according to the tables of accuracy using MAPE for the three banks, all models provided high performance because MAPE <10%.

In general, the empirical results of this study agree with earlier empirical works such as Abbas & Alhagyan (2022; 2023), Mansour & Alhagyan (2022), Alhagyan & Yassen (2023), Alhagyan (2022), Willinger et al. (1999), Painter (1998), Rejichi & Aloui (2012) and Alhagyan & Alduais (2020) to name just a few. Consequently, the researchers of this work strongly recommend that investors and speculators invest in the banking sector in Saudi Arabia because of its stability and predictability.

Use of AI Tools Declaration: The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

Funding: The authors extend their appreciation to Prince Sattam Bin Abdulaziz University for funding this research work through the project number (IF2/PSAU/ 2023/01/25099).

Data Availability Statement: The data that support the findings of this study are available on Saudi Exchange Market website at https://www.saudiexchange.sa

Author Contributions: Mustafa Mansour conceptualized, designed and collected data for the research study and analysed the data, wrote the Mathematical code and wrote the first draft of the manuscript and reviewed and approved the final version of the manuscript

REFERENCES

[1] Al-Sherman, N. (2024). Legal Protection from Artificial Intelligence Technology Used to Filter Visual Contents via the Internet. Pakistan Journal of Criminology, 16(1), 505–517. https://doi.org/10.62271/pjc.16.1.505.517 Legal Protection from Artificial Intelligence Technology Used to Filter Visual Contents via the Internet.pdf

[2] Al-Sherman, N. (2025). Smart Contracts: Methods of Documentation, Applications and Integration with Artificial Intelligence. The Arab World Geographer, 28(1), 49–61. https://doi.org/10.5555/1480-6800-28.1.49

[3] Abbas, A., & Alhagyan, M. (2022). The effect of incorporating memory and stochastic volatility into geometric Brownian motion in forecasting the performance of Tadawul all share Index (TASI). Advances and Applications in Statistics, 74, 47-62.

[4] Abbas, A., & Alhagyan, M. (2023). Forecasting Exchange Rate of Sar/Cny By Incorporating Memory and Stochastic Volatility Into GBM Model. Advances and Applications in Statistics, 86(1), 65-78.

[5] Alhagyan, M. (2022). The effects of incorporating memory and stochastic volatility into GBM to forecast exchange rates of Euro. Alexandria Engineering Journal, 61(12), 9601-9608.

[6] Alhagyan, M. K. M. (2018). Modeling financial environments using geometric fractional Brownian motion model with long memory stochastic volatility. Universiti Utara Malaysia.

[7] Alhagyan, M., & Alduais, F. (2020). Forecasting the performance of Tadawul all share index (TASI) using geometric Brownian motion and geometric fractional Brownian motion. Adv. Appl. Stat, 62(1), 55-65.

[8] Alhagyan, M., & Yassen, M. F. (2023). Incorporating stochastic volatility and long memory into geometric Brownian motion model to forecast performance of Standard and Poor's 500 index. AIMS Mathematics, 8(8), 18581-18595.

[9] Alhagyan, M., Misiran, M. & Omar, Z. (2016 a). Geometric fractional Brownian motion perturbed by fractional Ornstein-Uhlenbeck process and application on KLCI option pricing. Open Access Library Journal, 3(08), 1.

[10] Alhagyan, M., Misiran, M. & Omar, Z. (2021). "On effects of stochastic volatility and long memory towards mortgage insurance models: an empirical study", Advances and Applications in Statistics, vol. 66, no.2, pp. 165-174.

[11] Alhagyan, M., Misiran, M., & Omar, Z. (2016 b). Estimation of geometric fractional Brownian motion perturbed by stochastic volatility model. Far East Journal of Mathematical Sciences, 99(2), 221.

[12] Bardhan, A., Karapandža, R., & Urošević, B. (2006). Valuing mortgage insurance contracts in emerging market economies. The Journal of Real Estate Finance and Economics, 32(1), 9-20.

[13] Chen, C. C., Lin, S. K., & Chen, W. S. (2013). Mortgage insurance premiums and business cycle. Tunghai University working paper.

[14] Gozgor, G. (2013). The application of stochastic processes in exchange rate forecasting: Benchmark test for the EUR/USD and the USD/TRY. Economic Computation and Economic Cybernetics Studies and Research, 47 (2), 225-246.

[15] Grau–Carles, P. (2000). Empirical evidence of long–range correlations in stock returns. Physica A: Statistical Mechanics and its Applications, 287(3), 396-404.

[16] Han, C., Wang, Y., & Xu, Y. (2020). Nonlinearity and efficiency dynamics of foreign exchange markets: evidence from multifractality and volatility of major exchange rates. Economic research-Ekonomska istraživanja, 33(1), 731-751.

[17] Kim, K. H., Kim, N. U., Ju, D. C., & Ri, J. H. (2020). Efficient hedging currency options in fractional Brownian motion model with jumps. Physica A: Statistical Mechanics and its Applications, 539, 122868.

[18] Kukush, Mishura,Y., & Valkeila,E. (2005). Statistical inference with fractional brownian motion. Statistical Inference for Stochastic Processes, 8, 71-93.

[19] Misiran, M. (2010). Modeling and pricing financial assets under long memory processes (Doctoral dissertation). Curtin University of Technology.

[20] Misiran, M., Zudi, L.U., Teo, K. L., & Grace, A. W. (2012). Estimating dynamic geometric fractional Brownian motion and its application to long–memory option pricing. Dynamic Systems and Applications, 21(1), 49–66

[21] Painter, S. (1998). Numerical method for conditional simulation of Levy random fields. Mathematical Geology, 30(2), 163-179

[22] Rejichi, I. Z., & Aloui, C. (2012). Hurst exponent behavior and assessment of the MENA stock markets efficiency. Research in International Business and Finance, 26(3), 353–370

[23] Sheldon, M. R. (1999). An introduction to mathematical finance: Options and other topics.

[24] Wang, X., Xie, D., Jiang, J., Wu, X., & He, J. (2017). Value-at-Risk estimation with stochastic interest rate models for option-bond portfolios. Finance Research Letters, 21, 10-20.

[25] Willinger, W., Taqqu, M. S., & Teverovsky, V. (1999). Stock market prices and long– range dependence. Finance and Stochastics, 3(1), 1-13.

[26] Xiao, W., Zhang, W., & Zhang, X. (2015). Parameter identification for the discretely observed geometric fractional Brownian motion. Journal of Statistical Computation and Simulation, 85(2), 269-283