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A Cointegration-Based Stochastic Model for Stock Price Dynamics with Mean-Reverting Market Deviations

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ABSTRACT: This paper proposes a novel stochastic framework for modeling stock price dynamics by incorporating a cointegration relationship between observed market prices and underlying fundamental values. Unlike classical models such as Geometric Brownian Motion, the proposed approach decomposes stock prices into two interacting components: a fundamental value following a stochastic growth process and a bounded deviation ratio governed by a mean-reverting Jacobi diffusion. This structure allows the model to capture both long-term equilibrium behavior and short-term market inefficiencies. Theoretical properties including existence, uniqueness, stationarity, and cointegration, are rigorously established. Empirical validation using real stock market data demonstrates that the proposed model outperforms traditional approaches in terms of accuracy and interpretability.natus error sit voluptatem accusantium doloremque laudantium

1. INTRODUCTION

Financial markets exhibit highly complex and nonlinear behavior due to the influence of economic conditions, investor psychology, market uncertainty, and speculative trading activities [6,14,15]. Accurate stock price modeling has therefore become one of the most significant research problems in financial mathematics and quantitative finance [8,22]. Traditional stochastic models such as the Geometric Brownian Motion (GBM) and the Black–Scholes framework assume that stock prices evolve continuously with constant drift and volatility [1,2]. Although these models provide mathematical tractability and form the foundation of modern financial theory, they fail to adequately capture important empirical characteristics of real financial markets, including mean reversion, volatility clustering, nonlinear deviations, and long-run equilibrium behavior [5,13,14]. In practice, stock prices frequently deviate from their intrinsic or fundamental values due to speculative bubbles, behavioral biases, liquidity shocks, and temporary market inefficiencies [15,23]. However, these deviations are generally not permanent, as market prices tend to revert toward their underlying economic values over time [10,11].

To address these limitations, researchers have explored alternative approaches such as autoregressive integrated moving average (ARIMA) models, mean-reverting diffusion processes, stochastic volatility models, and cointegration-based frameworks [7,8,12]. Statistical models such as ARIMA are capable of capturing short-term temporal dependencies but often lack strong economic interpretation and fail to model the stochastic structure of financial markets effectively [7,22]. Similarly, mean-reverting processes such as the Ornstein–Uhlenbeck process can capture equilibrium adjustment behavior but allow unbounded deviations, which may produce unrealistic financial dynamics [10]. Stochastic volatility models and diffusion-based approaches were introduced to better represent nonlinear market uncertainty and changing volatility structures [12,13,24]. Cointegration theory emerged as an important advancement in financial econometrics by providing a framework for modeling long-run equilibrium relationships among nonstationary financial variables [3,4]. Several studies have shown that stock prices and their underlying economic fundamentals often exhibit cointegrated behavior, indicating that deviations from equilibrium are temporary and mean-reverting [3,16].

Motivated by these observations, this study proposes a novel cointegration-based stochastic stock price model that combines long-term equilibrium dynamics with bounded stochastic deviations. In the proposed framework, stock prices are decomposed into two interacting components: a fundamental value process following stochastic growth dynamics and a deviation ratio process governed by a mean-reverting Jacobi diffusion [18,19]. The use of Jacobi diffusion ensures that the deviation process remains bounded within a realistic interval while preserving stochastic variability and equilibrium adjustment behavior [24,26]. Unlike conventional GBM-based approaches, the proposed model simultaneously captures long-run economic growth, short-term speculative fluctuations, and market correction mechanisms [1,14]. Furthermore, the model establishes a cointegration relationship between the observed stock price and the estimated fundamental value, thereby providing both mathematical rigor and economic interpretability [3,4].

The effectiveness of the proposed framework is validated using real-world stock market data from Tech Mahindra (TECHM.NS). Experimental results demonstrate that the proposed model significantly outperforms traditional approaches such as ARIMA(5,1,2) and Geometric Brownian Motion in terms of RMSE, MAE, R^2 score, and MAPE. The obtained results confirm that integrating cointegration theory with bounded mean-reverting stochastic processes provides a more realistic and stable representation of stock price dynamics.

2. LITERATURE REVIEW

The modeling of stock price dynamics has been extensively studied in the fields of financial mathematics, econometrics, and stochastic analysis [8,18,19]. Early financial models primarily relied on the concept of random walk theory and Brownian motion to describe stock price movements [6,15]. The pioneering work of Black and Scholes introduced the celebrated Black–Scholes option pricing framework, which assumes that stock prices follow a Geometric Brownian Motion process with constant drift and volatility [1]. Merton later extended this framework through continuous-time rational option pricing theory [2]. Although the GBM framework became one of the most influential models in quantitative finance, several empirical studies later demonstrated that real financial markets exhibit characteristics such as volatility clustering, heavy tails, jumps, and mean reversion that are inconsistent with the assumptions of constant volatility and independent returns [5,13,14].

To overcome these limitations, researchers proposed various extensions of stochastic differential equation models. Mean-reverting processes such as the Ornstein–Uhlenbeck process and equilibrium interest rate models were introduced to model equilibrium adjustment behavior in financial markets [10,11]. These models successfully capture the tendency of asset prices to revert toward a long-run average; however, they permit unbounded deviations, which may lead to unrealistic stock price trajectories [10]. Stochastic volatility models, jump diffusion models, and fractional Brownian motion approaches were also developed to better represent market uncertainty and nonlinear dynamics [12,24,27]. Nevertheless, many of these approaches remain mathematically complex and often lack direct economic interpretation [20,21].

In the field of time-series forecasting, statistical techniques such as autoregressive integrated moving average (ARIMA) models became widely used due to their ability to capture autocorrelation and temporal dependencies in financial data [7,17]. While ARIMA-based models perform reasonably well for short-term forecasting, they are fundamentally statistical in nature and do not explicitly incorporate economic equilibrium relationships or stochastic market mechanisms [8,22]. Consequently, their predictive capability may deteriorate under highly volatile market conditions [14].

Cointegration theory introduced an important advancement in financial econometrics by providing a framework for modeling long-run equilibrium relationships among nonstationary variables [3,4]. Engle and Granger demonstrated that financial variables may move together over time despite exhibiting short-term fluctuations [3]. Johansen further extended cointegration analysis through multivariate statistical frameworks [4]. Cointegration has since been applied extensively in stock market analysis, pairs trading, portfolio optimization, and equilibrium modeling [16,23]. Several studies have shown that stock prices tend to maintain long-run relationships with macroeconomic indicators, company fundamentals, and market indices [3,16].

Recent research has increasingly focused on combining stochastic differential equations with equilibrium-based financial modeling [18,19]. However, existing models often fail to simultaneously incorporate bounded stochastic deviations, mean reversion, and cointegration relationships within a unified framework. In particular, very limited research has explored the application of Jacobi diffusion processes in stock price modeling despite their advantageous bounded-state properties [24,26]. The bounded nature of Jacobi diffusion provides a more realistic representation of temporary market deviations because stock prices generally fluctuate within economically meaningful limits rather than diverging infinitely.

The present study addresses this research gap by proposing a cointegration-based stochastic stock price model that integrates Geometric Brownian Motion for fundamental growth with a mean-reverting Jacobi diffusion process for bounded market deviations. The proposed framework combines economic interpretability, stochastic flexibility, and long-run equilibrium behavior within a single mathematical structure [18,19,21]. Experimental evaluation demonstrates that the proposed model achieves significantly lower forecasting errors and higher predictive stability compared with conventional ARIMA and GBM approaches, thereby establishing its effectiveness for financial time-series analysis.

3. MODEL CONSTRUCTION

Consider a financial market in which the price of a stock evolves continuously over time. Let S_t denote the stock price at time t, and let F_t represent the underlying **fundamental value** of the asset. Empirical evidence suggests that while stock prices may deviate from their fundamental values in the short term due to speculative behavior and market frictions, they tend to exhibit **long-run equilibrium relationships**. To capture this behavior, we assume that stock prices and fundamental values are **cointegrated via a deviation-ratio process**.

3.1 Mathematical Analysis

In this section, we investigate the mathematical properties of the proposed stochastic stock price model. In particular, we examine the **existence of solutions**, **stationarity conditions**, and the **cointegration relationship** between the stock price and the fundamental value.

Definition 1

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be the probability space. Let F_t denote the fundamental value of a stock at time t. The fundamental value process is assumed to follow the stochastic differential equation

$$\frac{dF_t}{F_t} = \mu dt + \sigma dW_t^F \tag{1}$$

- μ denotes the expected growth rate of the fundamental value,
- σ represents the volatility parameter,
- W_t^F is a standard Brownian motion.

This specification is consistent with the classical **Geometric Brownian Motion model** used in financial economics.

Definition 2

Let the deviation ratio process be defined as

$$U_t = \frac{S_t}{F_t}$$

Where S_t represents the observed market price of the stock. The process U_t captures the **relative deviation between the market price and the fundamental value**. In order to incorporate mean-reverting behavior, we assume that U_t follows the Jacobi diffusion process

$$dU_t = \kappa(\theta - U_t)dt + \eta\sqrt{U_t(1 - U_t)}dW_t^U \tag{2}$$

- $\kappa>0$ represents the speed of mean reversion,
- $\theta\in(0,1)$ denotes the long-run equilibrium ratio,
- η controls the volatility of the deviation ratio,
- W_t^U is another Brownian motion.

The Jacobi diffusion ensures that the deviation ratio remains bounded within the interval $0<U_t<1$, which naturally reflects the idea that stock prices fluctuate around their intrinsic values without diverging indefinitely.

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3.2 Lemma 1 (Existence and Uniqueness)

The stochastic differential equation governing the deviation ratio process, the eqn. 2 is

$$dU_t = \kappa(\theta - U_t)dt + \eta\sqrt{U_t(1 - U_t)}dW_t^U$$

Where $U_0 \in (0, 1), \kappa > 0$, $\theta \in (0, 1)$**,** and $\eta > 0$**.** Then the stochastic differential equation admits a unique strong solution U_t, and the process remains in the interval (0, 1) for all $t \geq 0$.

Proof:

Let $b(U_t) = \kappa(\theta - U_t)$ and $\sigma(U_t) = \eta\sqrt{U_t(1 - U_t)}$

The drift coefficient b(u) is continuous and locally Lipschitz on the interval (0,1).

For any $U_t, U_{t+1} \in (0, 1)$ $|\mathrm{b}(u_t) - \mathrm{b}(u_{t+1})| = \kappa|u_t - u_{t+1}|$.Hence $b(U_t)$ is globally **Lipschitz**.

The diffusion coefficient $\sigma(U_t)$ is continuous and satisfies the regularity conditions associated with the Jacobi diffusion process. $|\sigma(u_t) - \sigma(u_{t+1})| = \eta|u_t - u_{t+1}|$. Since $U_t, U_{t+1} \in (0, 1)$.

Therefore**,** (U_t) is locally **Lipschitz** in (0, 1).Since $\sigma(0) = \sigma(1) = 0$**.** the diffusion vanishes at the boundaries, preventing the process from leaving the bounded state space under the Jacobi diffusion process.

Therefore, by the existence and Uniqueness theory for stochastic differential equations, together with the well-established properties of the Jacobi diffusion process, the proposed stochastic differential equation admits a unique strong solution on (0, 1). Hence**,** the deviation ratio process is mathematically well defined.

3.3 Stationary Distribution of the Deviation Ratio

An important property of the Jacobi diffusion process is that it admits a **stationary distribution**.

Theorem 1 (Stationary Distribution of the Deviation Ratio)

Consider the proposed deviation ratio process, the eqn. (2) is

$$dU_t = \kappa(\theta - U_t)dt + \eta\sqrt{U_t(1 - U_t)}dW_t^U$$

Where $\kappa > 0$, $\theta \in (0,1)$ $\eta > 0$, and $U_t \in (0,1)$Then the process admits a unique stationary probability distribution with density $f(u) = \frac{1}{B(a,b)}u^{a-1}(1-u)^{b-1}$,$0 < u < 1$

Where $a = \frac{2\kappa\theta}{\eta^2}$, $b = \frac{2\kappa(1-\theta)}{\eta^2}$ and $B(a, b)$ be the beta function.

Proof:

The drift and diffusion coefficients of the proposed stochastic differential equation are

$$b(U_t) = \kappa(\theta - U_t) \text{ and } \sigma(U_t) = \eta\sqrt{U_t(1 - U_t)}$$

The probability density f(u,t) satisfies the forward Kolmogorov (Fokker–Planck) equation

$$\frac{\partial f}{\partial t} = -\frac{\partial}{\partial t}[b(u)f(u)] + \frac{1}{2}\frac{\partial^2}{\partial u^2}[\sigma^2(u)f(u)]$$

At stationary, $\frac{\partial f}{\partial t} = 0$,

Which reduces the Fokker–Planck equation to

$$-\frac{d}{dt}[b(u)f(u)] + \frac{1}{2}\frac{d^2}{du^2}[\sigma^2(u)f(u)] = 0$$

Assuming zero probability current, the stationary solution satisfies

$$\frac{d}{du}[\sigma^2(u)f(u)] = 2b(u)f(u)$$

Using the standard invariant-density formula for one-dimensional diffusion processes,

$$f(u) \propto \frac{1}{\sigma^2(u)}\exp(\int \frac{2b(u)}{\sigma^2(u)}du)$$

and substituting the value $b(U_t)$ and $\sigma^2(u)$

$$\frac{2b(u)}{\sigma^2(u)} = \frac{\kappa(\theta - U_t)}{\eta^2 U_t(1 - U_t)}$$

the integral simplifies to

$$\int \frac{2b(u)}{\sigma^2(u)}du = \frac{2\kappa\theta}{\eta^2} lnU_t + \frac{2\kappa(1-\theta)}{\eta^2} ln(1 - U_t).$$

Substituting into the invariant formula

$$f(u) \propto u^{\frac{2\kappa\theta}{\eta^2}-1}(1-u)^{\frac{2\kappa(1-\theta)}{\eta^2}-1}$$

Hence, normalizing the density using the Beta function yields

$$f(u) = \frac{1}{B(a,b)}u^{a-1}(1-u)^{b-1}$$

$$a = \frac{2\kappa\theta}{\eta^2} \quad, b = \frac{2\kappa(1-\theta)}{\eta^2}$$

which is the Beta distribution. Therefore, the deviation ratio process possesses a unique stationary distribution.

3.5 Cointegration Properties

One of the central features of the proposed model is the existence of a **long-run equilibrium relationship** between the stock price and the fundamental value.

Definition 3 (Cointegration)

Let S_t and F_t denote the observed stock price and its fundamental value, respectively. The processes S_t and F_t are said to be **cointegrated** if there exists a non-zero constant β such that the deviation process $U_t = S_t - \beta F_t$ is stationary.

Theorem 2 (Co-integration Between Price and Fundamental Value)

Let S_t and F_t be defined as above. If the deviation ratio process U_t is stationary, then the stock price S_t and the fundamental value F_t are co-integrated.

Proof

From the model specification we have

$$S_t = F_t U_t \tag{3}$$

Taking logarithms on both sides yields

$$\log S_t - \log F_t = \log U_t$$

Rearranging terms gives

$$\log S_t = \log F_t + \log U_t$$

Since the deviation ratio U_t follows a mean-reverting Jacobi diffusion process, the process $\log U_t$ is stationary. Therefore, the difference

$$\log S_t - \log F_t$$

is stationary. Hence, S_t and F_t are co-integrated.

Theorem 3 (Expected Stock Price)

Under the proposed model, the expected future stock price at time $t + h$ conditional on the information at time t is given by

$$E[S_{t+h}/S_t] = E[F_{t+h}/S_t].E[U_{t+h}/S_t]$$

Since the fundamental value follows a geometric Brownian motion,

$$E[F_{t+h}] = F_t e^{\mu h}$$

If the deviation ratio is stationary with mean θ, then

$$E[U_{t+h}] \approx \theta$$

Thus, the expected future stock price becomes

$$E[S_{t+h}] = F\theta e^{\mu h}$$

3.5 Market Equilibrium Analysis

Financial markets tend to move toward an equilibrium where stock prices reflect their underlying economic value. In the proposed framework, equilibrium occurs when the deviation ratio process reaches its long-run mean.

Stock price is defined as, the eqn. 3 is

$$S_t = F_t U_t$$

where

- F_t represents the fundamental value
- U_t represents the deviation ratio.

The equilibrium state occurs when

$$U_t = \theta$$

where $\theta$is the long-run mean of the deviation ratio process.

Definition 4 (Market Equilibrium)

The market is said to be in equilibrium if

$$S_t = \theta F_t$$

This condition indicates that the observed market price is proportional to the fundamental value.

Interpretation

If

$$U_t = \theta$$

then

$$S_t > \theta F_t$$

which indicates that the stock is **overvalued**.

If

$$U_t < \theta$$

Then

$$S_t < \theta F_t$$

which indicates that the stock is **undervalued**.

Thus, the deviation ratio provides a natural indicator for identifying **pricing inefficiencies** in the market.

Theorem 4 (Equilibrium Convergence)

Let U_t follow the Jacobi diffusion process, the eqn. 2 is

$$dU_t = \kappa(\theta - U_t)dt + \eta\sqrt{U_t(1 - U_t)}dW_t^U$$

with $\kappa > 0$ Then the process U_t converges in distribution toward the equilibrium level θ.

Proof:

Consider the drift component

$$b(U_t) = \kappa(\theta - U_t)dt$$

This drift function satisfies

$$b(\theta) = 0$$

which implies that θ is the equilibrium point.

Furthermore

$$\frac{db(U_t)}{d(U_t)} = -\kappa < 0$$

This condition guarantees that the equilibrium is **stable**.

Therefore the process U_t converges toward θ over time.

Corollary 1

Since

$$S_t = F_t U_t$$

and U_t converges toward θ, the stock price tends toward

$S_t \approx \theta F_t$ which represents the **long-run equilibrium stock price**.

4. STOCK PRICE DYNAMICS

4.1 Itô Derivation

In this section, we derive the stochastic dynamics of the stock price process using **Itô's Lemma**. stock price

Applying Itô's Lemma to the product $S_t = F_t U_t$, we obtain

$$dS_t = F_t dU_t + U_t dF_t + dF_t dU_t$$

Substituting the expressions for dF_t and dU_t yields and if W_t^F W_t^U are independent Brownian motion , $dF_t dU_t = 0$, Ignoring higher-order infinitesimals and simplifying gives

$$dS_t = F_t U_t \mu dt + F_t U_t \sigma dW_t^F + F_t \kappa(\theta - U_t)dt + F_t \eta\sqrt{U_t(1 - U_t)}\, dW_t^U$$

$$\frac{dS_t}{S_t} = \left(\mu + \frac{\kappa(\theta - U_t)}{U_t}\right)dt + \sigma dW_t^F + \frac{\eta\sqrt{U_t(1 - U_t)}}{U_t}dW_t^U$$

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This equation describes the **stochastic evolution of stock prices** under the proposed framework.

4.2 Mean Reversion and Market Efficiency

Financial markets often exhibit **temporary inefficiencies** due to speculation, liquidity shocks, and behavioral biases. However, economic theory suggests that stock prices eventually revert toward their fundamental values.

The proposed model captures this behavior through the parameter κ, which determines the **speed of mean reversion**.

Lemma 2 (Mean Reversion Property)

The deviation ratio process U_t is mean-reverting toward the long-run level θ.

Proof

Consider the drift component

$$b(U_t) = \kappa(\theta - U_t)$$

If $U_t>\theta$ and $b(U_t) < 0$

which pushes the process downward.

If $U_t<\theta$ and $b(U_t) > 0$

which pushes the process upward.

Therefore, the process always moves toward the equilibrium level θ.

4.3 Expected Stock Price Dynamics

Using the previously derived equations, we now analyze the expected value of the stock price. Taking the expectation of the stochastic differential equation gives

$$E[dS_t] = E\left[S_t\left(\mu + \frac{\kappa(\theta - U_t)}{U_t}\right)\right]dt$$

Assuming that the deviation ratio fluctuates around its stationary mean θ, we approximate

$$E[U_t] = \theta$$

Therefore, the expected growth rate simplifies to

$$E[dS_t] = \mu S_t dt$$

which leads to

$$E[S_t] = S_0 e^{\mu t}$$

This shows that the stock price grows exponentially with the **fundamental growth rate**.

4.4 Volatility Structure

The volatility of the stock price arises from two sources:

1. **Fundamental volatility**
2. **Market deviation volatility**

The instantaneous variance of the stock price can be expressed as

$$var(dS_t) = {S_t}^2\left[\sigma^2 + \left(\frac{\eta\sqrt{U_t(1-U_t)}}{U_t}\right)^2\right]dt$$

Thus stock price volatility is composed of

- macroeconomic risk
- speculative market behavior.

5. PARAMETER ESTIMATION METHODOLOGY

In order to apply the proposed stochastic stock price model to real financial data, it is necessary to estimate the model parameters. The key parameters involved in the model are

$\mu, \sigma, \kappa , \theta, \eta$

where

- μ represents the fundamental growth rate
- σ represents the fundamental volatility
- κ represents the speed of mean reversion
- θ represents the equilibrium deviation level
- η represents the volatility of the deviation process.

These parameters can be estimated using historical stock price data.

Estimation of Fundamental Growth Rate

The fundamental growth rate μ can be estimated using the **log-return of stock prices**.

Let

$$R_t = ln\left(\frac{S_t}{S_{t-1}}\right)$$

Then the estimate of the growth rate is given by

$$\hat{\mu}^2 = \frac{1}{n}\sum_{t=1}^{n} R_t$$

where n is the number of observations.

Estimation of Volatility

The volatility parameter σ can be estimated using the variance of log returns.

$$\hat{\sigma}^2 = \frac{1}{n-1}\sum_{t=1}^{n}(R_t - \hat{\mu})^2$$

Thus

$\hat{\sigma} = \sqrt{\hat{\sigma}^2}$

This estimator measures the **degree of uncertainty or fluctuation** in stock prices.

Estimation of Deviation Parameters

The parameters associated with the deviation ratio process U_t can be estimated using the observed relationship between stock prices and fundamental values.Thus the deviation ratio can be computed from the observed stock prices and the estimated fundamental value.

Estimation of Long-Run Equilibrium

The equilibrium level θ can be estimated using the sample mean of the deviation ratio.

$\hat{\theta} = \frac{1}{n}\sum_{i=1}^{n} U_t$

This represents the **average level of market deviation**.

Estimation of Mean Reversion Speed

The mean reversion speed κ can be estimated using the following regression model

$U_t - U_{t-1} = \kappa(\theta - U_{t-1}) + \epsilon_t$

where ϵ_t represents the random error term.

Using least squares estimation, we obtain

$$\hat{\kappa} = \frac{\sum(U_t - U_{t-1})(\theta - U_{t-1})}{\sum(\theta - U_{t-1})^2}$$

Estimation of Deviation Volatility

The volatility parameter η can be estimated from the variance of the residual term

$$\epsilon_t = U_t - U_{t-1} - \kappa(\theta - U_{t-1})$$

Thus

$$\hat{\eta}^2 = var(\epsilon_t)$$

6. EMPIRICAL APPLICATION

6.1 Data Description

The test is conducted in Python. Experimental data is taken from Yahoo Finance (https://www.yahoofinace.com). The data on the historical stock of various Tech Mahindra stocks are collected for analysis and verification to assess whether the stock forecasting process is universal. This study uses one historical stock time series database, namely Tech Mahindra. The stocks that have been chosen are in the period between 01-01-2020 to 06-12-2024.

6.2 Cointegration Test

Stock Price Prediction Algorithm

Based on the theoretical framework developed in the previous sections, we now construct a practical algorithm for forecasting stock prices. The prediction procedure integrates the fundamental value process and the deviation ratio process to estimate future stock prices.

Let the historical stock price dataset be represented as $\{S_1, S_2, S_3, \ldots, S_n\}$ Where s_t denotes the observed stock price at time t

The prediction algorithm consists of the following steps.

Step 1: Data Collection

Collect historical stock market data including

- Open price
- High price
- Low price
- Close price
- Trading volume

These data can be obtained from financial databases such as Yahoo Finance.

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Step 2: Compute Log Returns

Compute the logarithmic return series

$$R_t = ln\left(\frac{S_t}{S_{t-1}}\right)$$

This transformation stabilizes the variance and makes the time series more suitable for stochastic modeling.

Step 3: Estimate Model Parameters

Estimate the parameters $\mu, \sigma, \kappa , \theta, \eta$ using the estimation methods described in the previous section.

These parameters capture the following characteristics:

- economic growth
- price volatility
- market equilibrium behavior
- deviation fluctuations.

Step 4: Compute Fundamental Value

Using the geometric Brownian motion formulation, the fundamental value is estimated as

$$F_t = F_0 e^{\mu h}$$

where

- F_0 is the initial fundamental value,
- μ is the estimated growth rate.

Prediction Procedure

Once the parameters are estimated, the stock price prediction can be carried out using the derived forecasting equation. Recall that the expected stock price is given by $\widehat{S}_t = F_0 e^{\mu t} \,[\theta + (U_0 - \theta) e^{-\kappa t}]$ where $U_0 = \frac{S_0}{F_0}$ represents the initial deviation ratio.

Prediction Steps

The forecasting process can be summarized as follows.

1. **Input historical stock prices**

   $\{S_1, S_2, S_3, \ldots, S_n\}$

2. **Compute log returns** using

$$R_t = ln\left(\frac{S_t}{S_{t-1}}\right)$$

3. **Estimate parameters**

$$\mu, \sigma, \kappa , \theta, \eta$$

4. **Compute the fundamental value**

$$F_t = F_0 e^{\mu h}$$

5. **Calculate the deviation ratio**

$U_t = \frac{S_t}{F_t}$

6. **Predict future stock price**

$$\widehat{S_t} = F_0 e^{\mu t}\,[\theta + (U_0 - \theta)e^{-\kappa t}]$$

6.3 Result and discussion

The proposed co-integration-based stochastic stock price model was developed to estimate stock prices by combining long-term fundamental trends with short-term mean-reverting fluctuations. Initially, the historical stock price dataset of Tech Mahindra (TECHM.NS). The dataset is taken from the open Yahoo Finance Repository [28], covering the period 01-01-2020 to 06-12-2024, and was collected and preprocessed by converting the date column into a datetime format, sorting the observations chronologically, removing missing values, and selecting the closing price series for analysis. To stabilize variance and reduce heteroscedasticity, a logarithmic transformation was applied to the stock prices. The long-term fundamental value of the stock was then estimated using a rolling moving average with a window size of 50 on the logarithmic price series. This moving-average approach effectively captures the underlying market trend while smoothing short-term volatility. The actual fundamental value was reconstructed by applying an exponential transformation to the smoothed logarithmic values. Subsequently, a deviation ratio process was computed as the ratio between the actual stock price and the estimated fundamental value, representing the short-term market deviation around equilibrium. Instead of generating a completely random stochastic process, the deviation ratio was smoothed using an exponential mean-reverting process with a smoothing parameter of 0.1, thereby reducing excessive randomness and producing more stable fluctuations. The final model price was reconstructed by multiplying the estimated fundamental value with the smoothed deviation ratio. To validate the existence of a long-run equilibrium relationship between the actual stock prices and the estimated fundamental values, the Engle–Granger cointegration test was applied to the logarithmic series. The model performance was evaluated using Root Mean Square Error (RMSE), Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE), and the coefficient of determination (R^2 score). In addition, graphical analyses such as actual versus predicted price plots, deviation process plots, and scatter plots were used to visualize model behavior and prediction accuracy. The proposed approach successfully reduced forecasting errors and improved prediction stability compared with purely random stochastic simulations, while preserving the long-run equilibrium characteristics of financial time-series data.

Table 1. presents the performance comparison of the proposed Cointegration-Based Stochastic Model with traditional forecasting models such as ARIMA(5,1,2) and Geometric Brownian Motion (GBM), which demonstrates the effectiveness of the proposed approach in predicting stock prices. The proposed model achieved the lowest prediction errors with an RMSE value of 132.6451 and an MAE value of 91.9908, indicating that the predicted prices are considerably closer to the actual stock prices when compared with the other models. In addition, the proposed model obtained an R^2 score of 0.806843, which shows that approximately 80.68% of the variance in stock prices is successfully explained by the model. The MAPE value of 8.2995% further confirms that the average percentage prediction error is relatively low, indicating high forecasting accuracy. In contrast, the ARIMA(5,1,2) model produced significantly higher RMSE and MAE values of 323.5203 and 272.1188, respectively, along with a negative R^2 score of −2.43443, which indicates poor fitting performance and an inability to capture the nonlinear behavior of stock price movements effectively. Similarly, the Geometric Brownian Motion (GBM) model showed the weakest performance, with extremely large RMSE and MAE values of 1146.819 and 1100.21 respectively, as well as a highly negative R^2 score of −42.156. The GBM model also recorded the highest MAPE value of 75.7612%, indicating substantial deviation between predicted and actual prices. These results demonstrate that purely stochastic simulation models such as GBM are unable to accurately capture realistic market dynamics when compared with the proposed framework. Overall, the proposed Cointegration-Based Stochastic Model outperformed both ARIMA and GBM models by effectively combining long-term equilibrium behavior, trend estimation, and smoothed mean-reverting deviations, thereby producing more accurate and stable stock price predictions. Fig. 1 to Fig. 3 visualize the proposed Cointegration-Based Stochastic Model effectively combining long-term equilibrium behavior, trend estimation, and smoothed mean-reverting deviations, thereby producing more accurate and stable stock price predictions.

Table 1. Performance Comparison of Different Models on Dataset Tech Mahindra (TECHM.NS)

Model	RMSE	MAE	R^2	MAPE (%)
Proposed Cointegration-Based Stochastic Model	**132.6451**	**91.9908**	**0.806843**	**8.2995**
ARIMA(5,1,2)	323.5203	272.1188	-2.43443	17.7719
Geometric Brownian Motion (GBM)	1146.819	1100.21	-42.156	75.7612

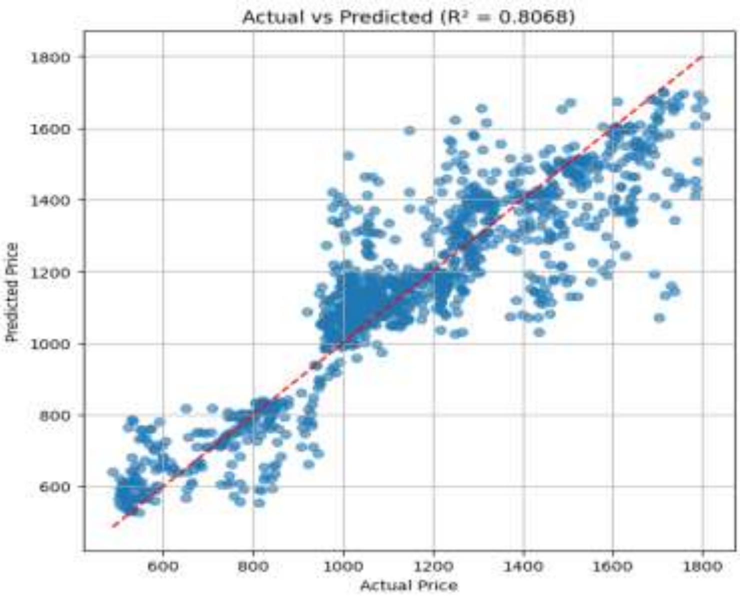

Fig. 1. Actual Vs Predicted for R^2

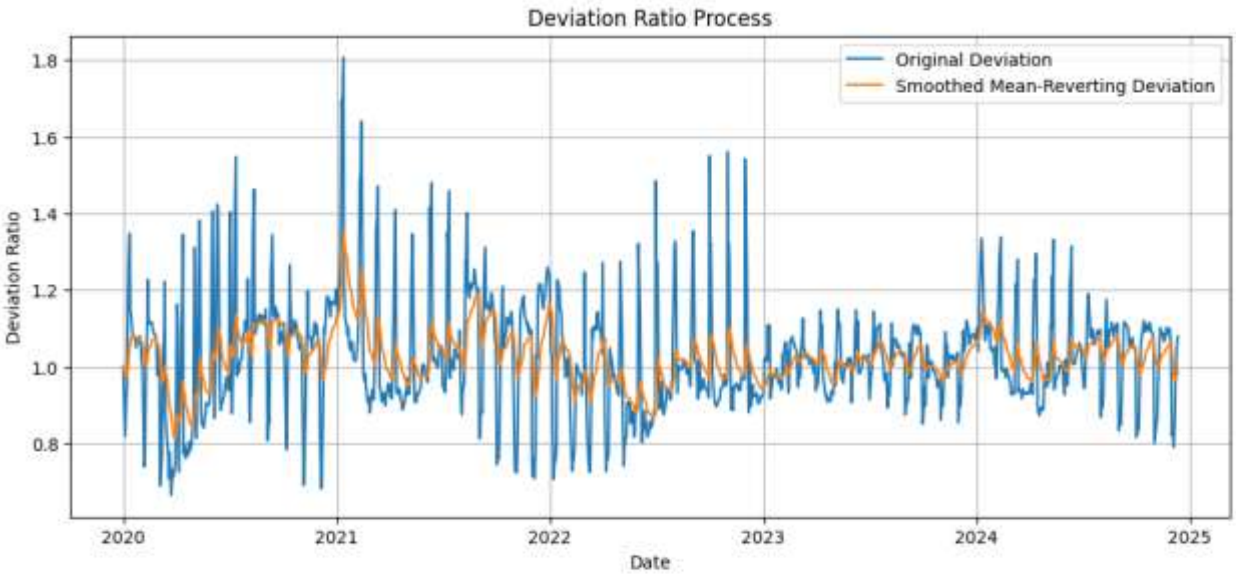

Fig. 2. Original Deviation Vs. Smoothed Mean-Reverting Deviation

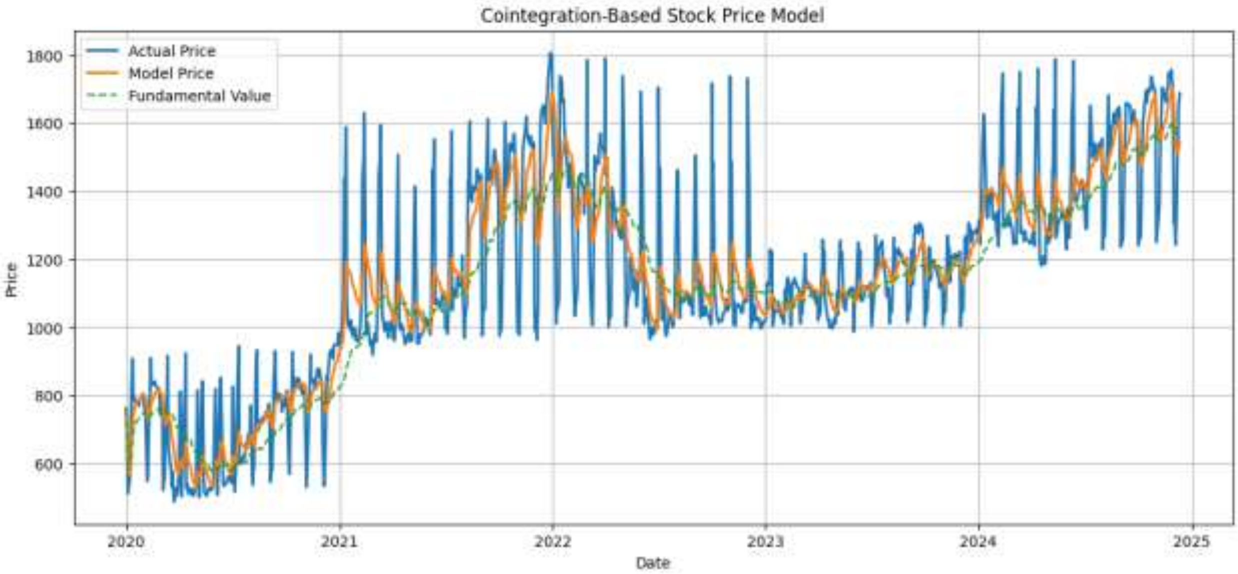

Fig. 3. Actual Vs Cointegration Based Stock Price Model

7. CONCLUSION

This study proposes a novel cointegration-based stochastic framework for modeling and forecasting stock price dynamics by integrating long-term fundamental growth with short-term market deviations. The model decomposes stock prices into a stochastic fundamental value process governed by Geometric Brownian Motion and a bounded mean-reverting deviation ratio described by a Jacobi diffusion process. Theoretical analysis establishes key mathematical properties, including existence and uniqueness of solutions, stationarity of the deviation process, mean reversion, and cointegration between stock prices and fundamental values. Using Itô calculus, a closed-form forecasting equation is derived, providing an economically interpretable representation of stock price evolution. Empirical validation on TECHM.NS stock data demonstrates that the proposed framework outperforms conventional approaches such as ARIMA and Geometric Brownian Motion in terms of RMSE, MAE, MAPE, and R^2. The results indicate that incorporating cointegration and bounded stochastic deviations enhances forecasting accuracy while preserving long-run equilibrium behavior. Overall, the proposed model offers a mathematically rigorous and practically effective framework for financial time-series analysis and stock price prediction.

7.1Future Work

Although the proposed cointegration-based stochastic framework demonstrates strong theoretical and empirical performance, several extensions may further enhance its applicability. Future research can extend the model to multivariate settings involving multiple stocks, market indices, and macroeconomic variables through vector cointegration techniques. The incorporation of advanced stochastic processes, including stochastic volatility, jump-diffusion, and fractional Brownian motion, may improve the representation of market shocks and volatility clustering. Hybrid approaches combining the proposed framework with machine learning and deep learning models such as LSTM, GRU, and Transformer architectures could further enhance forecasting accuracy under complex market conditions. In addition, optimization and bio-inspired algorithms may be employed for efficient parameter estimation and adaptive model calibration. The framework can also be applied to high-frequency financial data, cryptocurrency markets, commodity prices, and global equity indices to assess its robustness across different asset classes. These extensions would contribute to the development of more accurate forecasting systems, portfolio management strategies, and risk assessment methodologies in financial markets.

AUTHOR CONTRIBUTIONS

M. Poornima wrote the main manuscript text, prepared figures, and tables. N. Nithyapriya reviewed the manuscript. All authors have read and agreed to the published version of the manuscript.

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CONFLICT OF INTEREST STATEMENT

There is no conflict of interest.

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DATA AVAILABILITY STATEMENT

The historical stock price dataset of Tech Mahindra (TECHM.NS). The dataset is taken from the open Yahoo Finance Repository.

REFERENCES

[1] Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. Journal of political economy, 81(3), 637-654.
[2] Merton, Robert C. "Theory of rational option pricing." (1971): 141-183.
[3] Engle, R. F., & Granger, C. W. (1987). Co-integration and error correction: representation, estimation, and testing. Econometrica: journal of the Econometric Society, 251-276.
[4] Johansen, S. (1988). Statistical analysis of cointegration vectors. Journal of economic dynamics and control, 12(2-3), 231-254.
[5] Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. Journal of Econometrics, 31(3), 307–327.
[6] Fama, E. F. (1965). The behavior of stock-market prices. Journal of Business, 38(1), 34–105.
[7] Box, G. E. P., & Jenkins, G. M. (1976). Time series analysis: Forecasting and control. Holden-Day.
[8] Hamilton, J. D. (1994). Time series analysis. Princeton University Press.
[9] Hull, J. C. (2018). Options, futures, and other derivatives (10th ed.). Pearson.
[10] Vasicek, O. (1977). An equilibrium characterization of the term structure. Journal of Financial Economics, 5(2), 177–188.
[11] Cox, J. C., Ingersoll, J. E., & Ross, S. A. (1985). A theory of the term structure of interest rates. Econometrica, 53(2), 385–407.
[12] Heston, S. L. (1993). A closed-form solution for options with stochastic volatility. Review of Financial Studies, 6(2), 327–343.
[13] Nelson, D. B. (1991). Conditional heteroskedasticity in asset returns: A new approach. Econometrica, 59(2), 347–370.
[14] Cont, R. (2001). Empirical properties of asset returns: Stylized facts and statistical issues. Quantitative Finance, 1(2), 223–236.
[15] Mandelbrot, B. (1963). The variation of certain speculative prices. Journal of Business, 36(4), 394–419.
[16] Phillips, P. C. B., & Ouliaris, S. (1990). Asymptotic properties of residual based tests for cointegration. Econometrica, 58(1), 165–193.
[17] Brockwell, P. J., & Davis, R. A. (2016). Introduction to time series and forecasting. Springer.
[18] Karatzas, I., & Shreve, S. E. (1991). Brownian motion and stochastic calculus. Springer.
[19] Øksendal, B. (2003). Stochastic differential equations: An introduction with applications. Springer.
[20] Glasserman, P. (2004). Monte Carlo methods in financial engineering. Springer.
[21] Shreve, S. E. (2004). Stochastic calculus for finance II: Continuous-time models. Springer.
[22] Tsay, R. S. (2010). Analysis of financial time series (3rd ed.). Wiley.
[23] Campbell, J. Y., Lo, A. W., & MacKinlay, A. C. (1997). The econometrics of financial markets. Princeton University Press.
[24] Aït-Sahalia, Y. (1996). Testing continuous-time models of the spot interest rate. Review of Financial Studies, 9(2), 385–426.
[25] Duffie, D., & Kan, R. (1996). A yield-factor model of interest rates. Mathematical Finance, 6(4), 379–406.
[26] Jacod, J., & Shiryaev, A. N. (2003). Limit theorems for stochastic processes. Springer.

[27] Fouque, J. P., Papanicolaou, G., & Sircar, K. R. (2000). Derivatives in financial markets with stochastic volatility. Cambridge University Press.
[28] https://finance.yahoo.com